



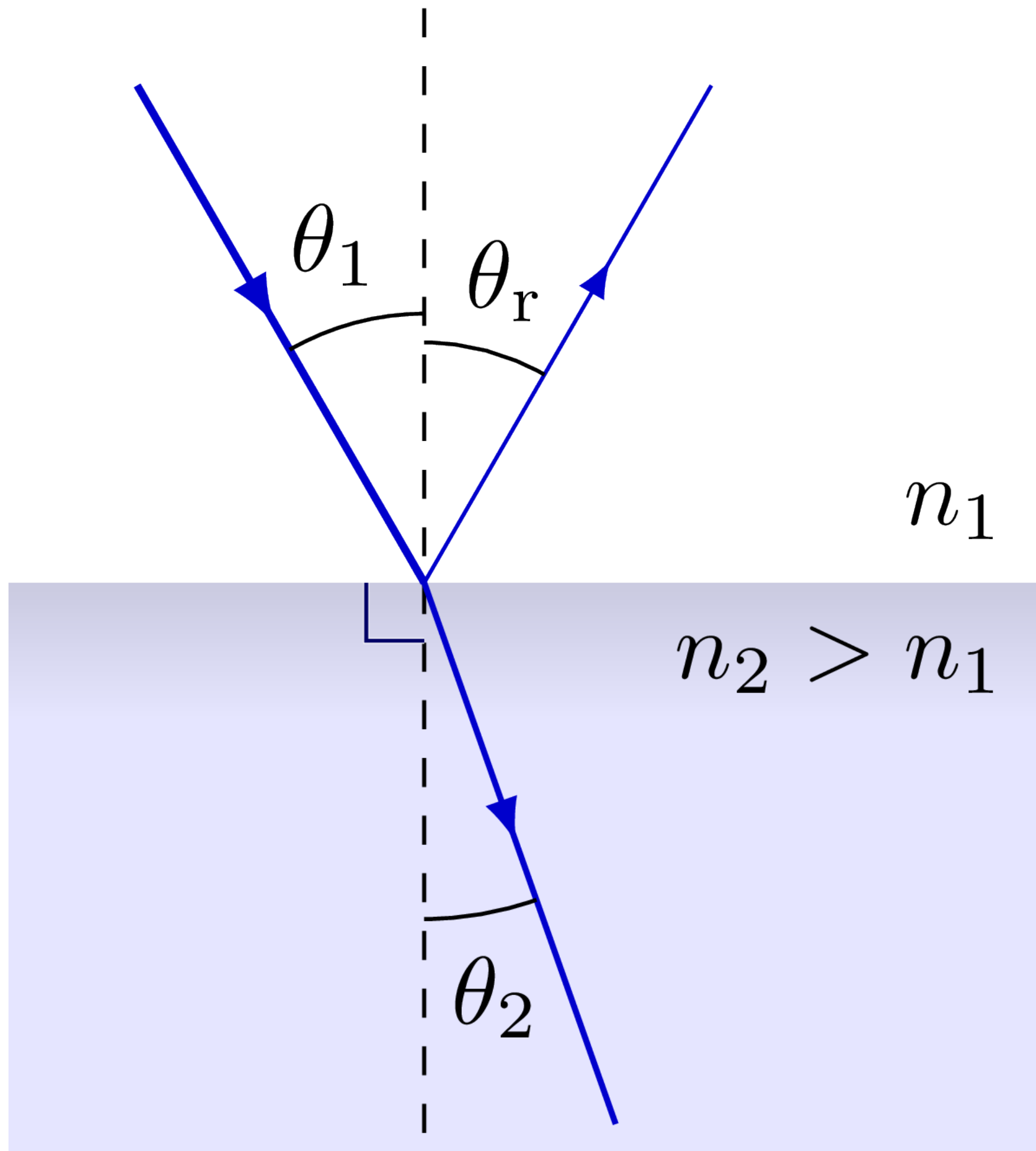
# Нелинейная и топологическая Фотоника

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**Москва 2024**

# Закон Снеллиуса



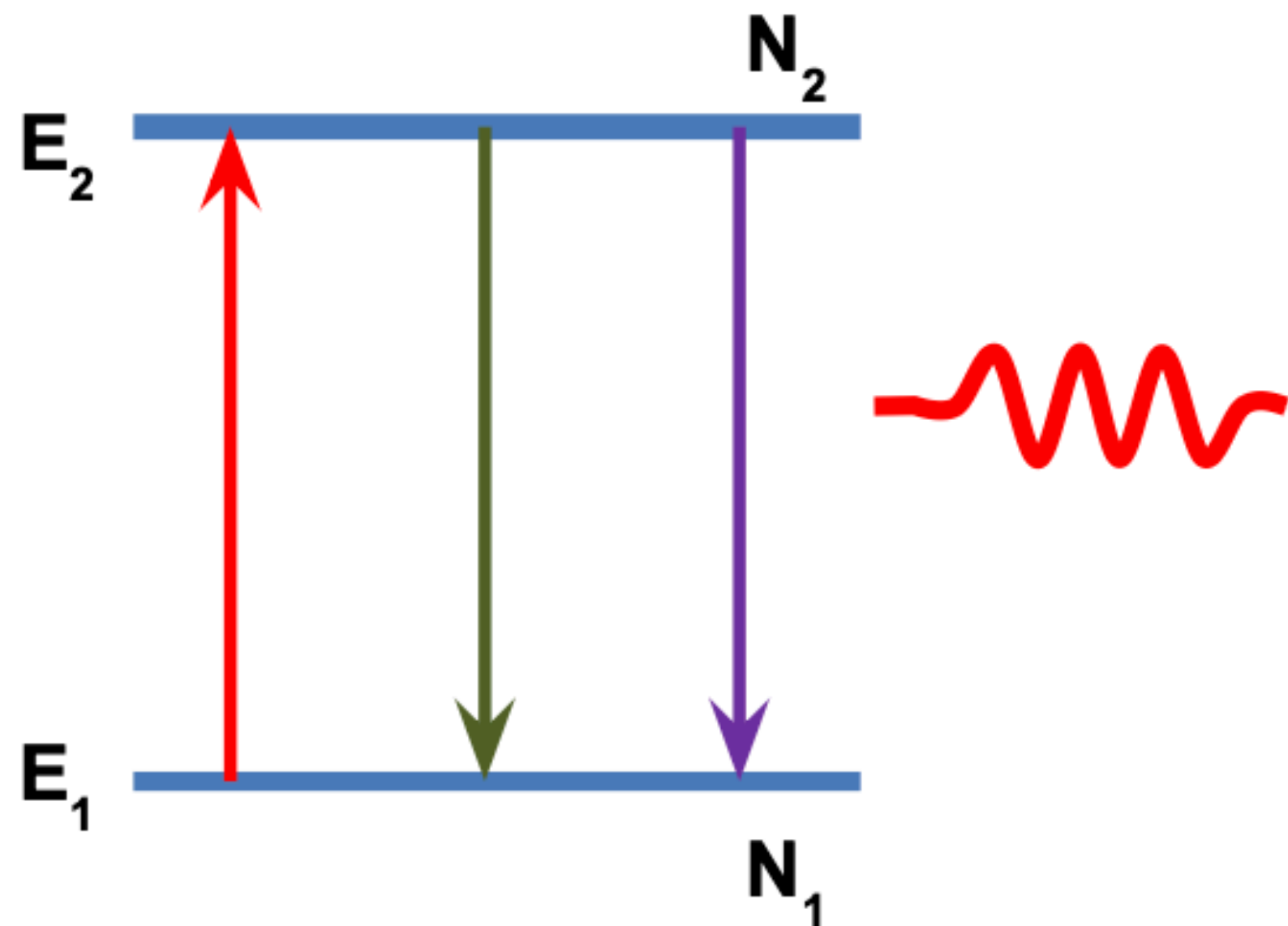
$$n_1 \sin \theta = n_2 \sin \theta_2$$

$$n = \sqrt{\mu \epsilon}$$

$\mu = 1$  — Для оптических материалов

Показатель преломления зависит  
только от свойств материала!

# Откуда берется показатель преломления?



Модель Лоренца

$$m_e \left( \frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x \right) = e E_0 e^{i\omega t}$$

$$n = 1 + \frac{N e^2}{2 \epsilon m_e (\omega_0^2 - \omega^2)}$$

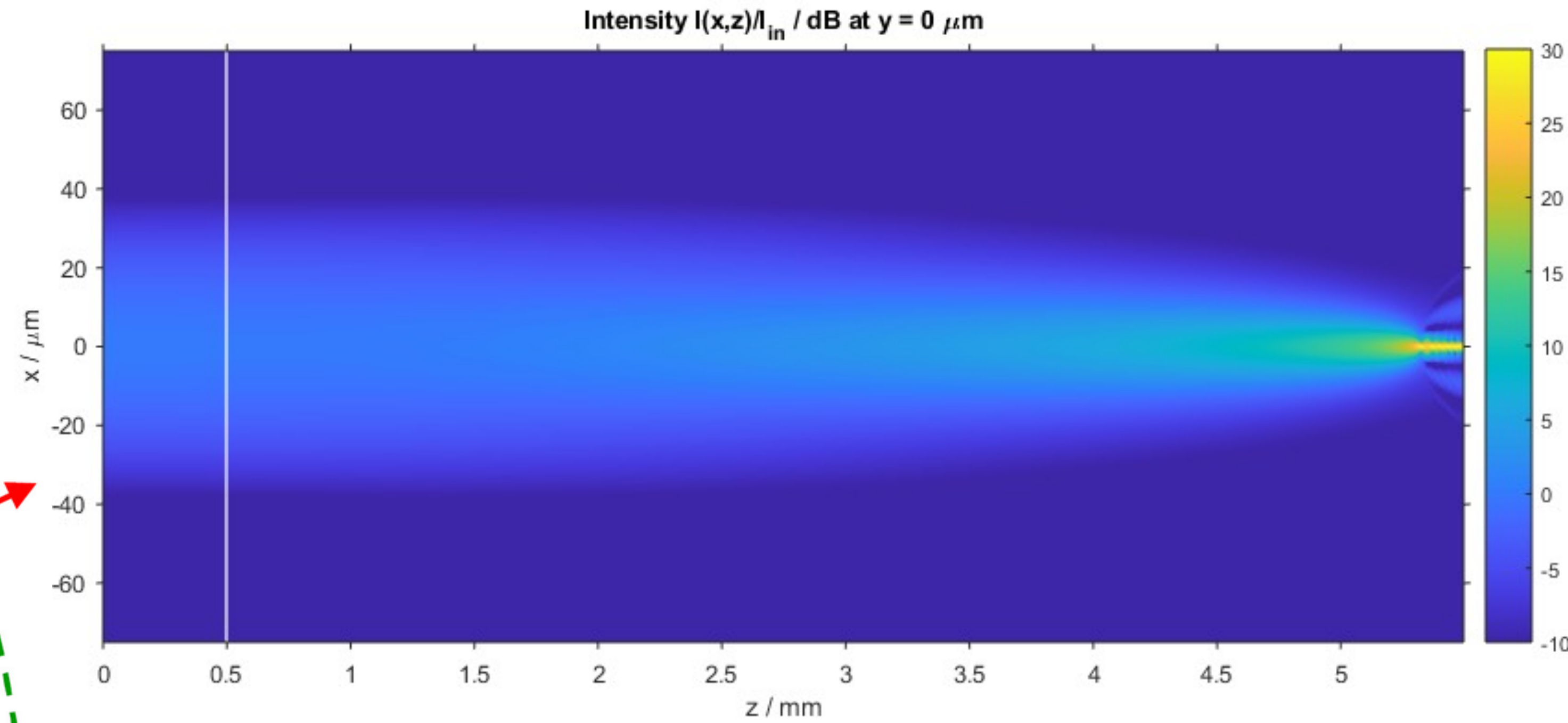
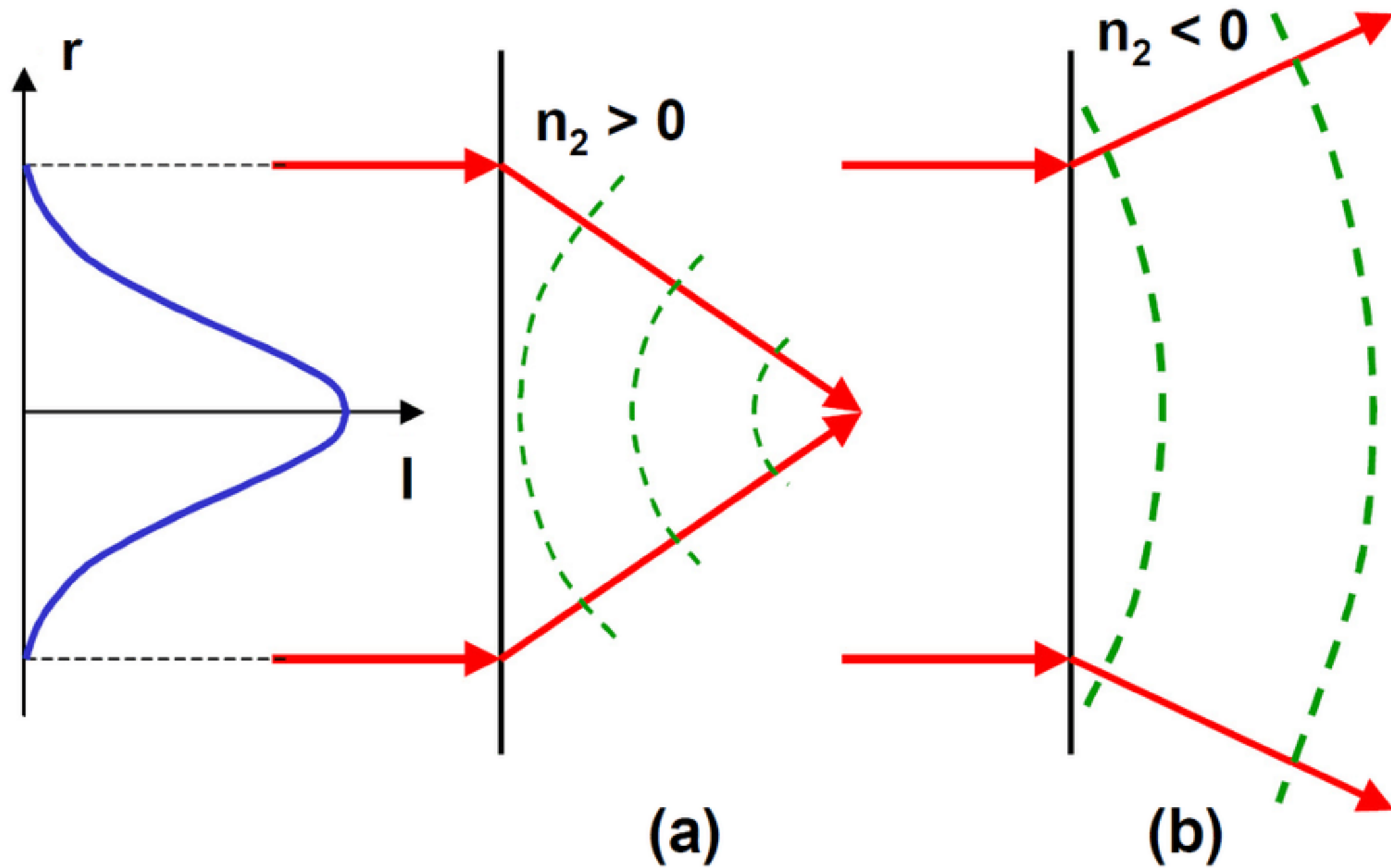
$$P = \epsilon_0 \int_0^t \chi(t - t') E(t') dt'$$

Показатель преломления зависит  
только от свойств материала!

**Может ли зависеть от  
поля?**

# Керровские материалы

$$n = n_0 + n_2 |E|^2$$



$$n_2 = 10^{-20} \text{m}^2 \text{W}^{-1}$$

# Вывод НУШ с кубической нелинейностью

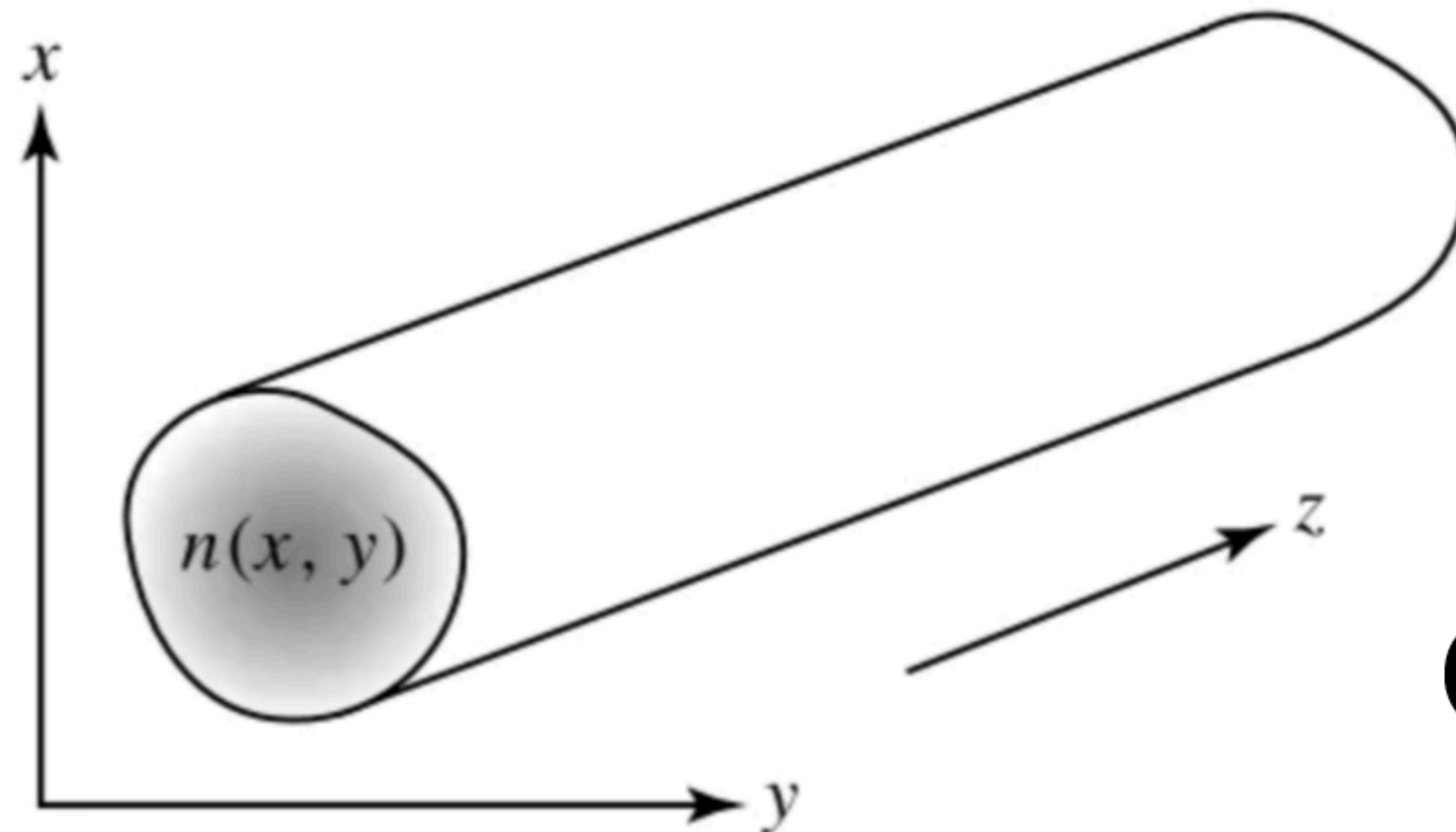
$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 P}{\partial t^2}$$

~~$$P_i = \epsilon_0 \int \chi_{ij}^{(1)} E_j(r, t - t_1) + \epsilon_0 \iint \chi_{ijk}^{(2)} E_j(r, t - t_1) E_k(r, t - t_2) + \epsilon_0 \iiint \chi_{ijkl}^{(3)} E_j(r, t - t_1) E_k(r, t - t_2) E_l(r, t - t_3)$$~~

$$x \rightarrow -x, \quad E \rightarrow -E, \quad P \rightarrow -P$$

$$E(r, t) = \frac{1}{2} E(r) [\exp[-i\omega t] + c.c.]$$

$$E(r) = A(r) \exp(i\beta_0 Z)$$



$$\ddot{\ddot{O}} = Si = \ddot{\ddot{O}}$$

# Нелинейное уравнение Шредингера

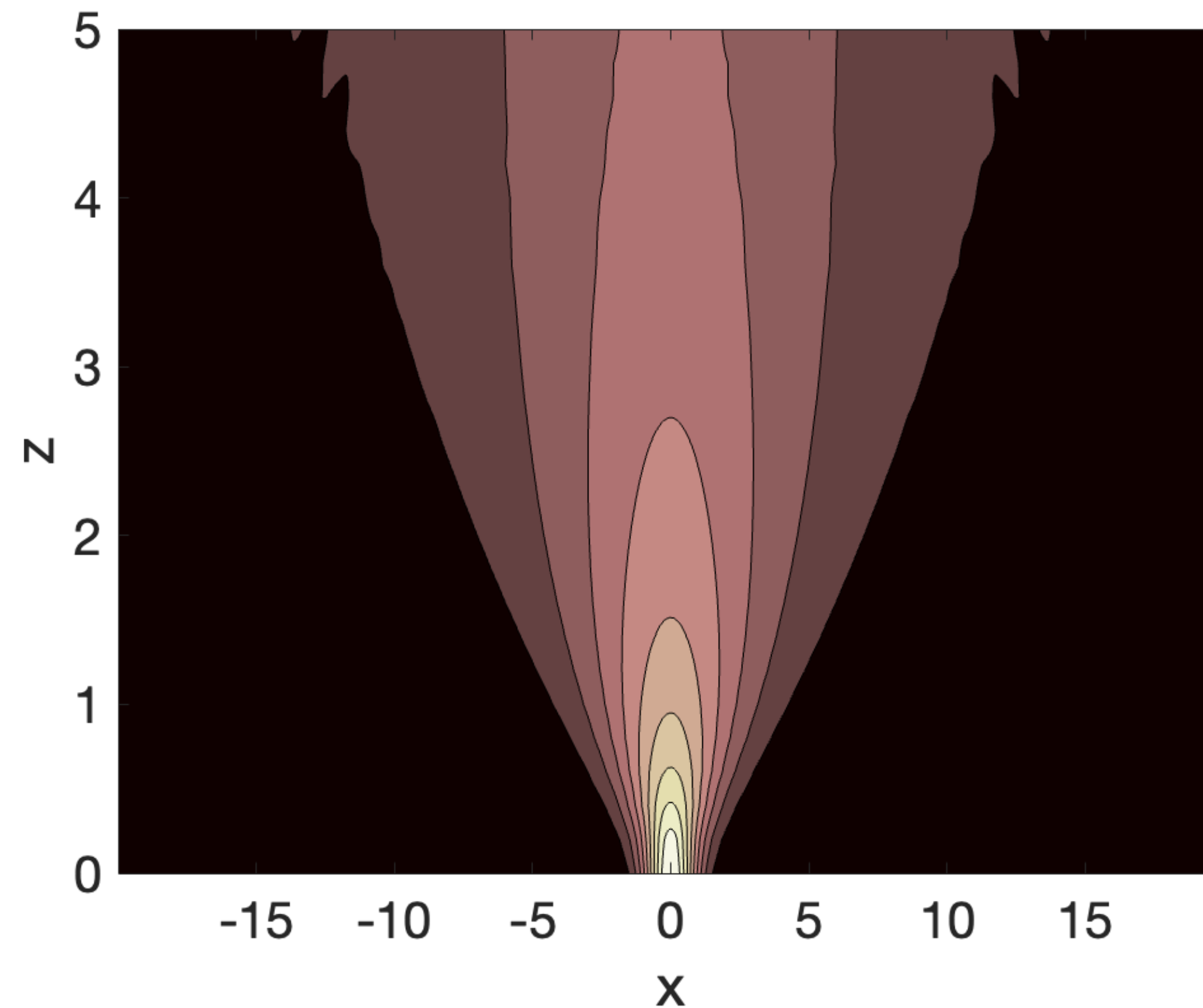
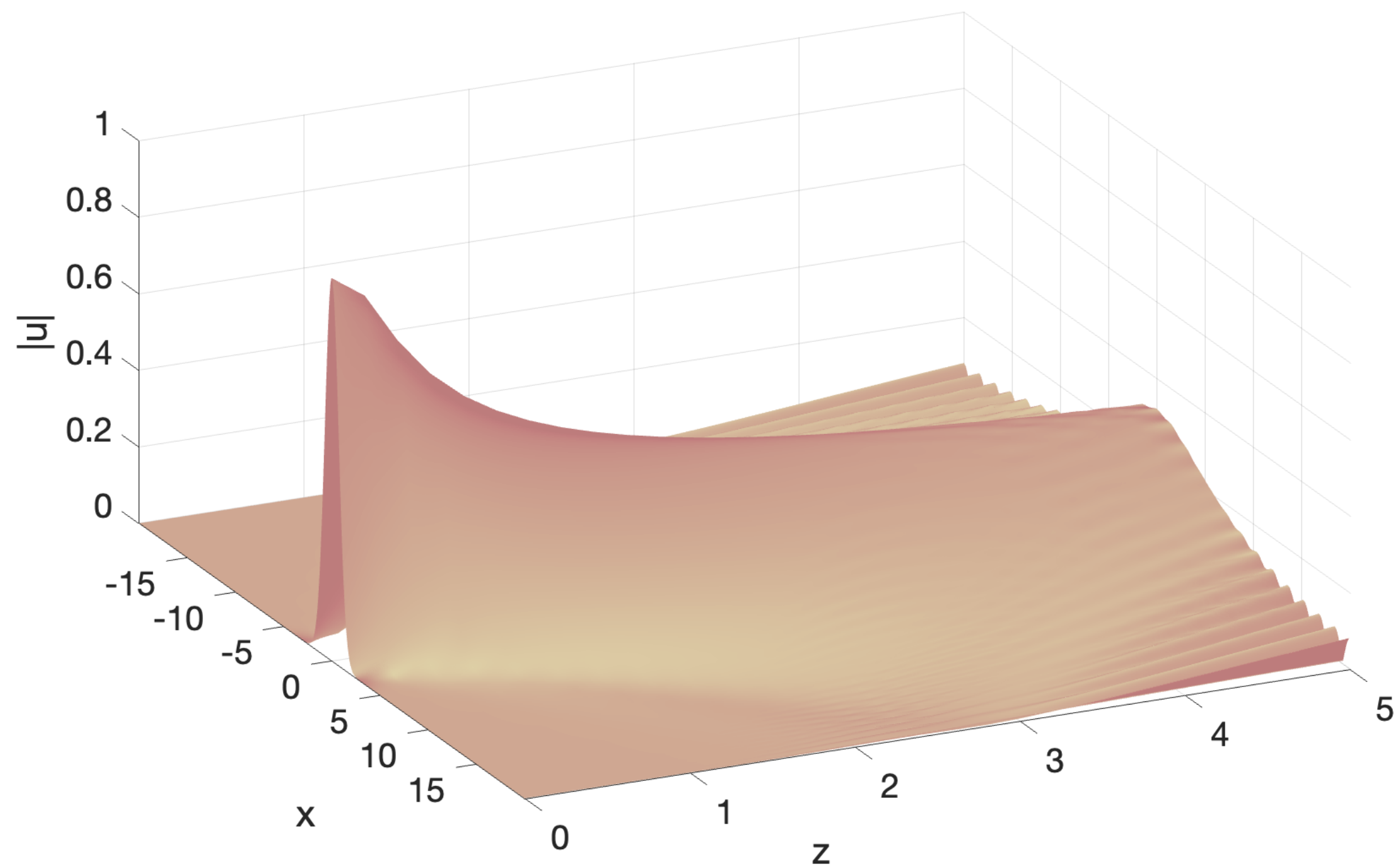
$$x = \frac{X}{\omega_0}, \quad y = \frac{Y}{\omega_0}, \quad z = \frac{Z}{L_d}, \quad u = (k_0 n_2 L_d)^{1/2} A$$

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

$$i \frac{\partial u}{\partial z} = -\frac{1}{2} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \sigma |u|^2 u$$

$$i\hbar \frac{\partial \psi}{\partial t} = \left( \frac{\hbar^2}{2m} \nabla^2 + U(r) \right) \psi$$

# Дисперсия пучка



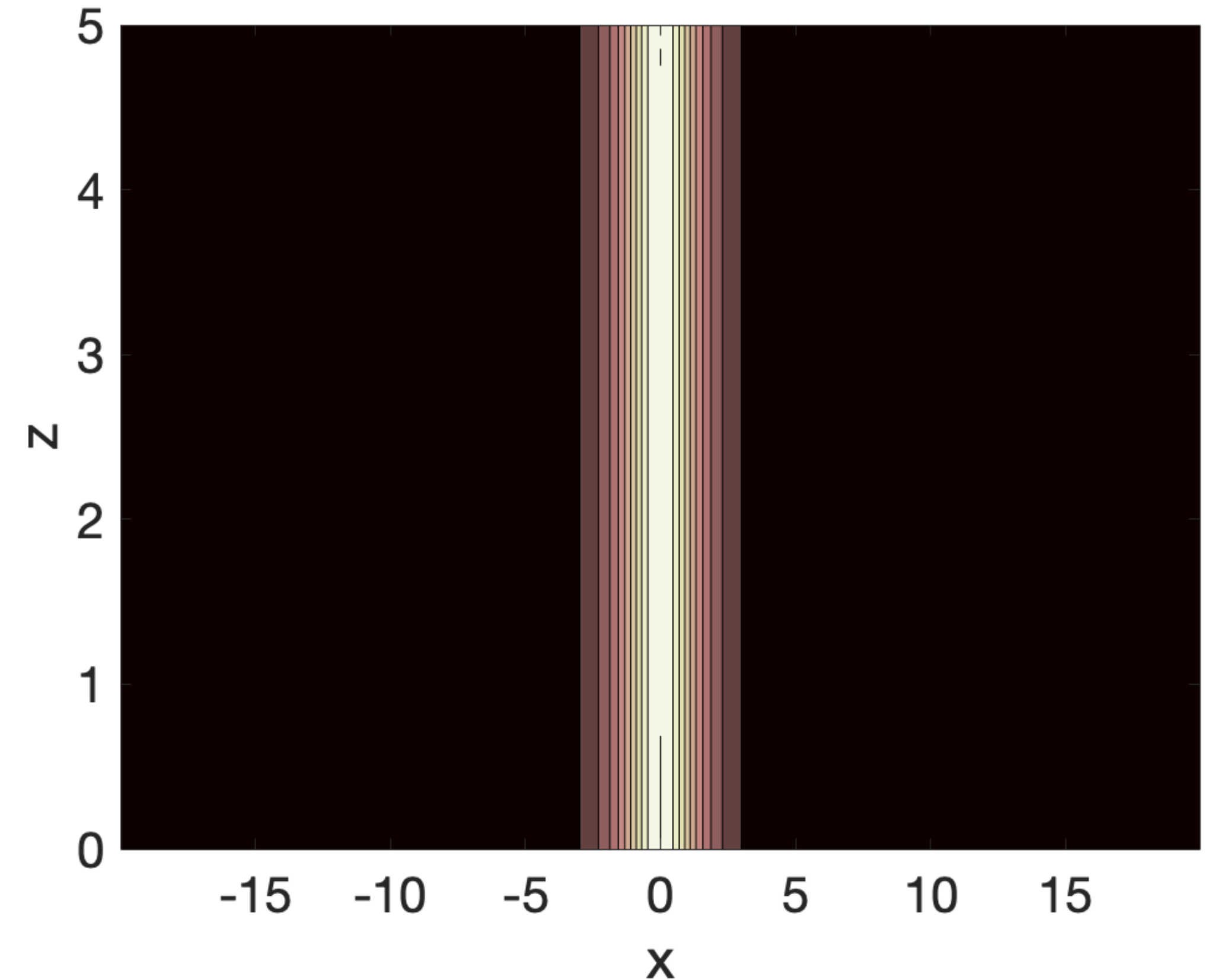
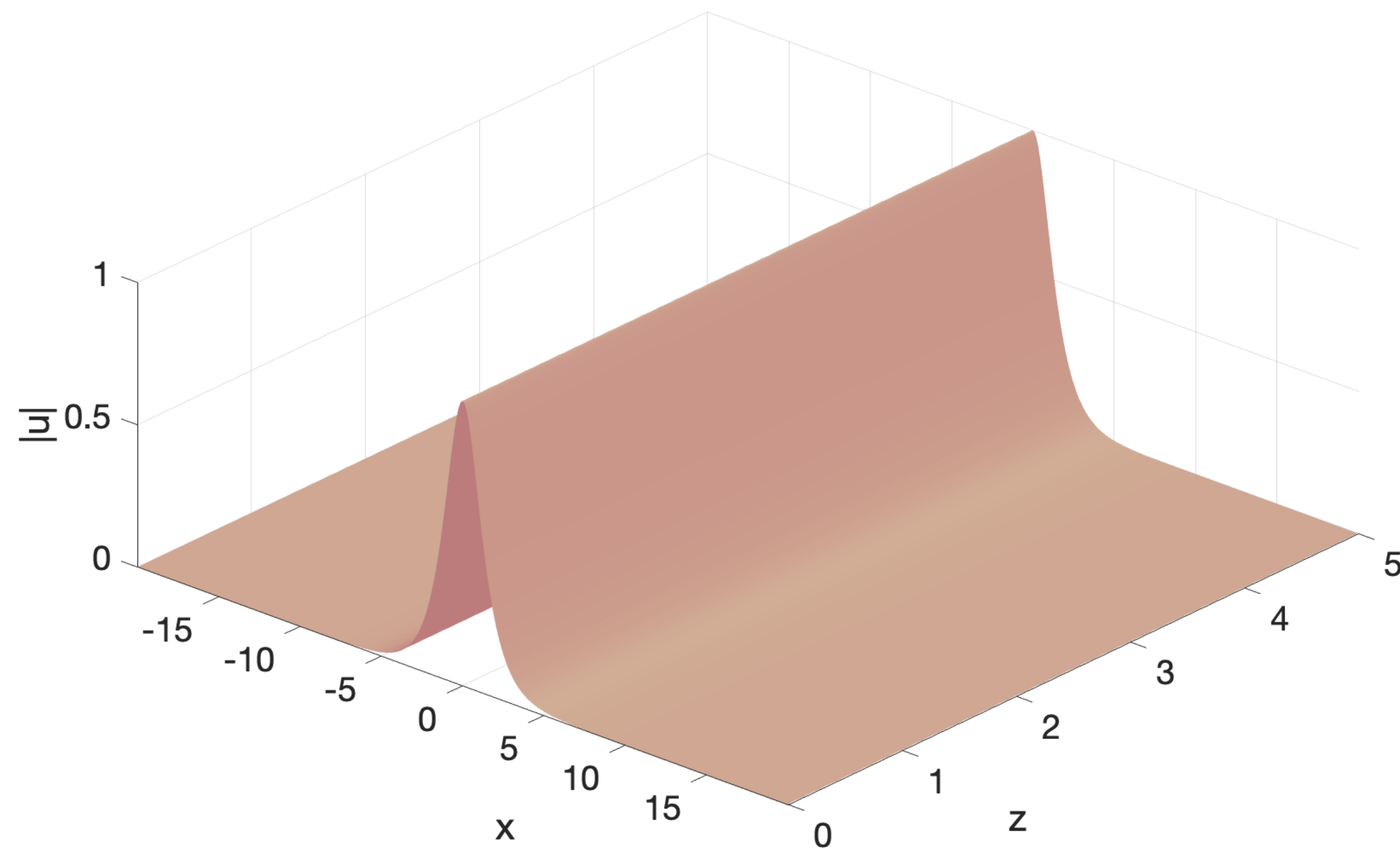
$$i \frac{\partial u}{\partial z} = - \frac{1}{2} \frac{\partial^2 u}{\partial x^2}$$



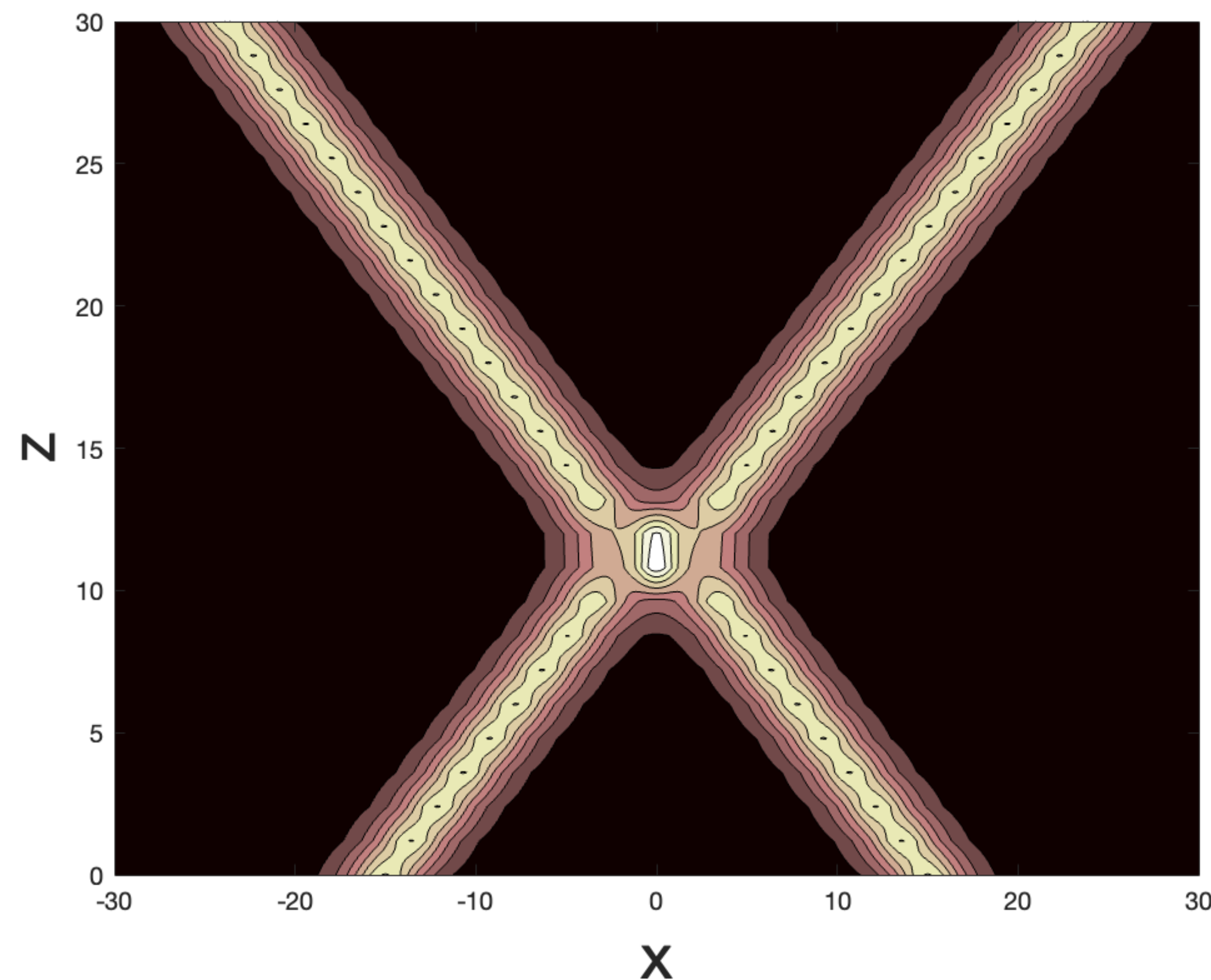
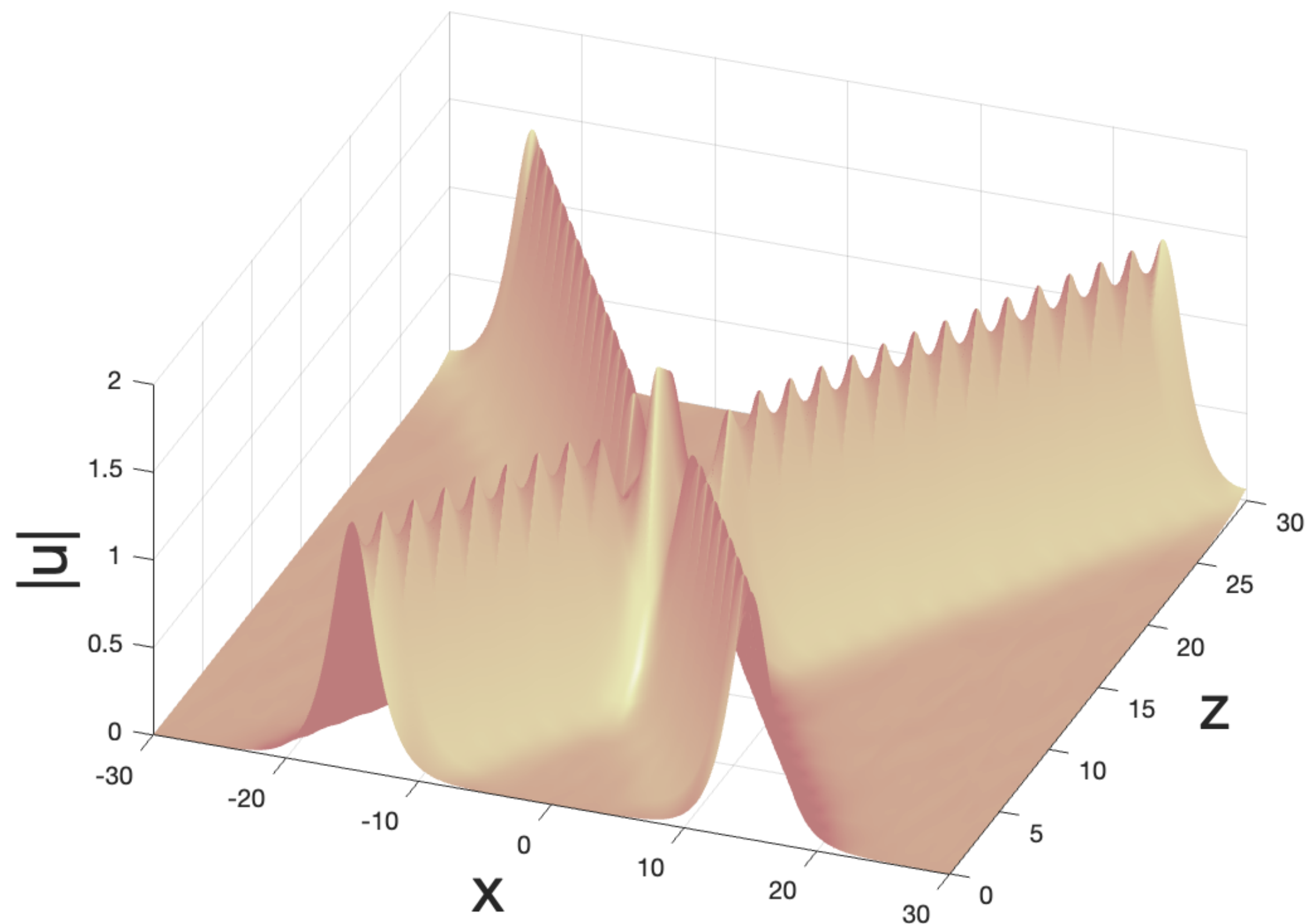
# В игру вступает нелинейность

$$u(x, z) = A \operatorname{sch}(\kappa x) e^{i\beta z}$$

$$i \frac{\partial u}{\partial z} = -\frac{1}{2} \frac{\partial^2 u}{\partial x^2} + \sigma |u|^2 u$$

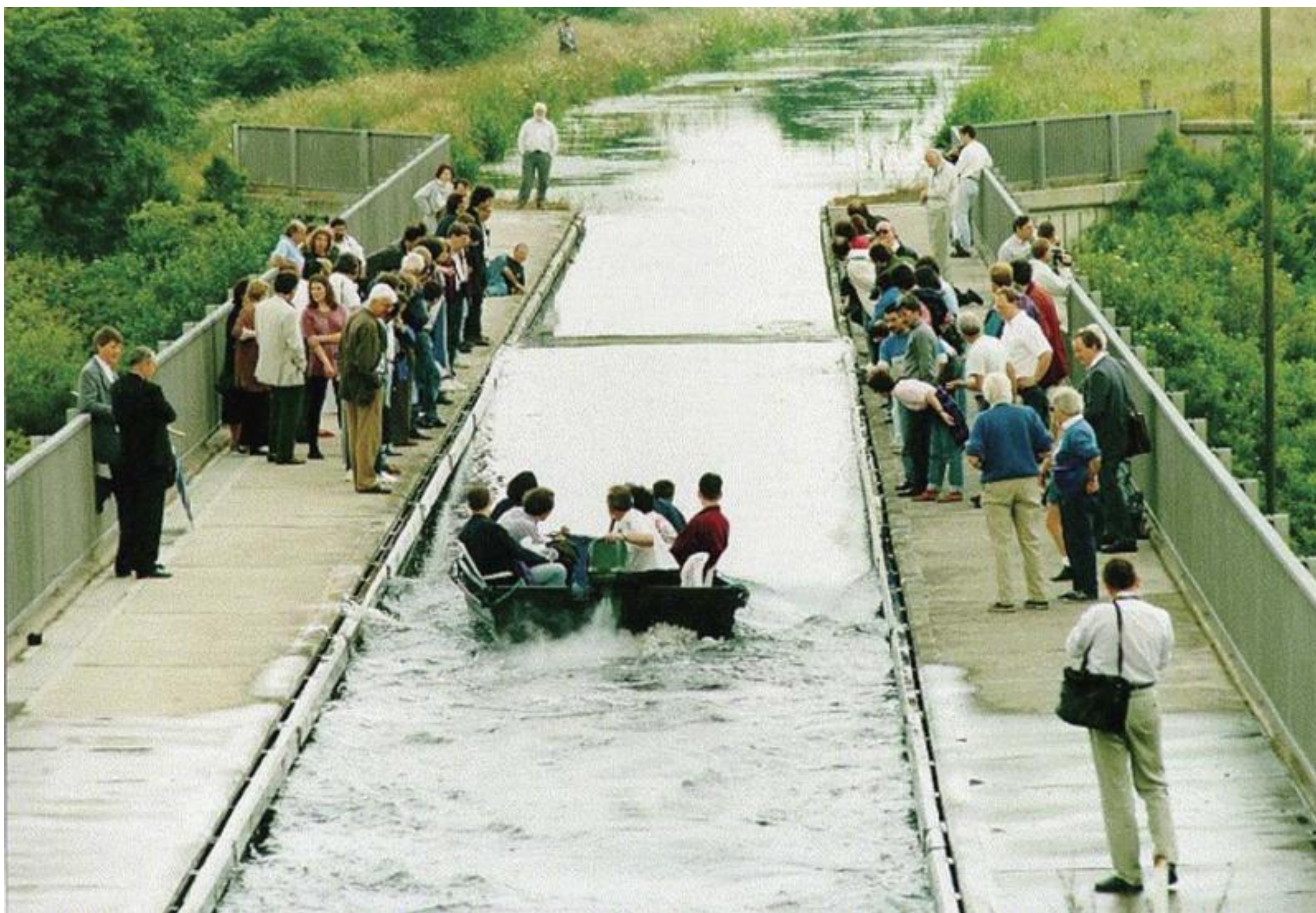


# Свойства солитонов

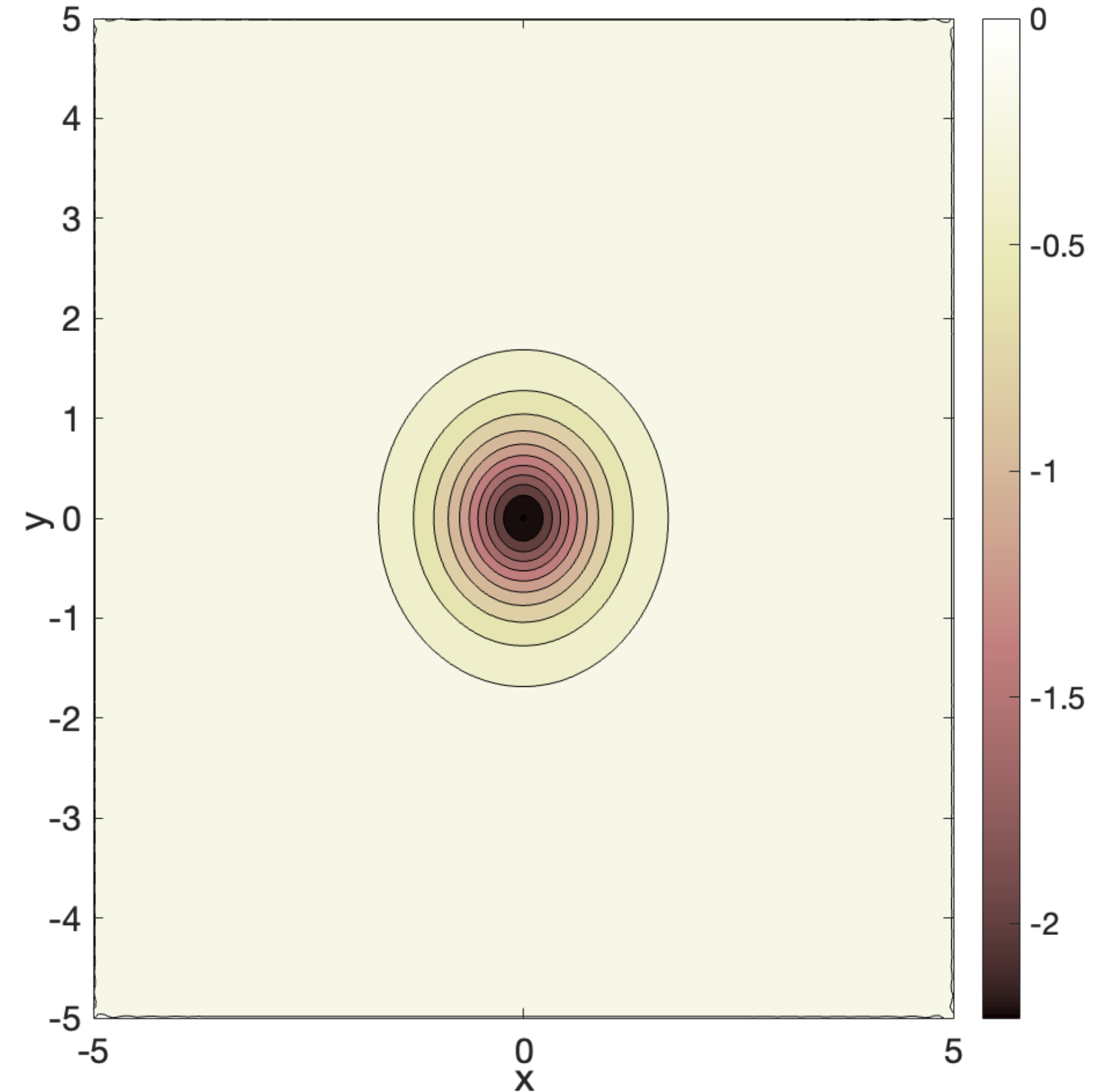
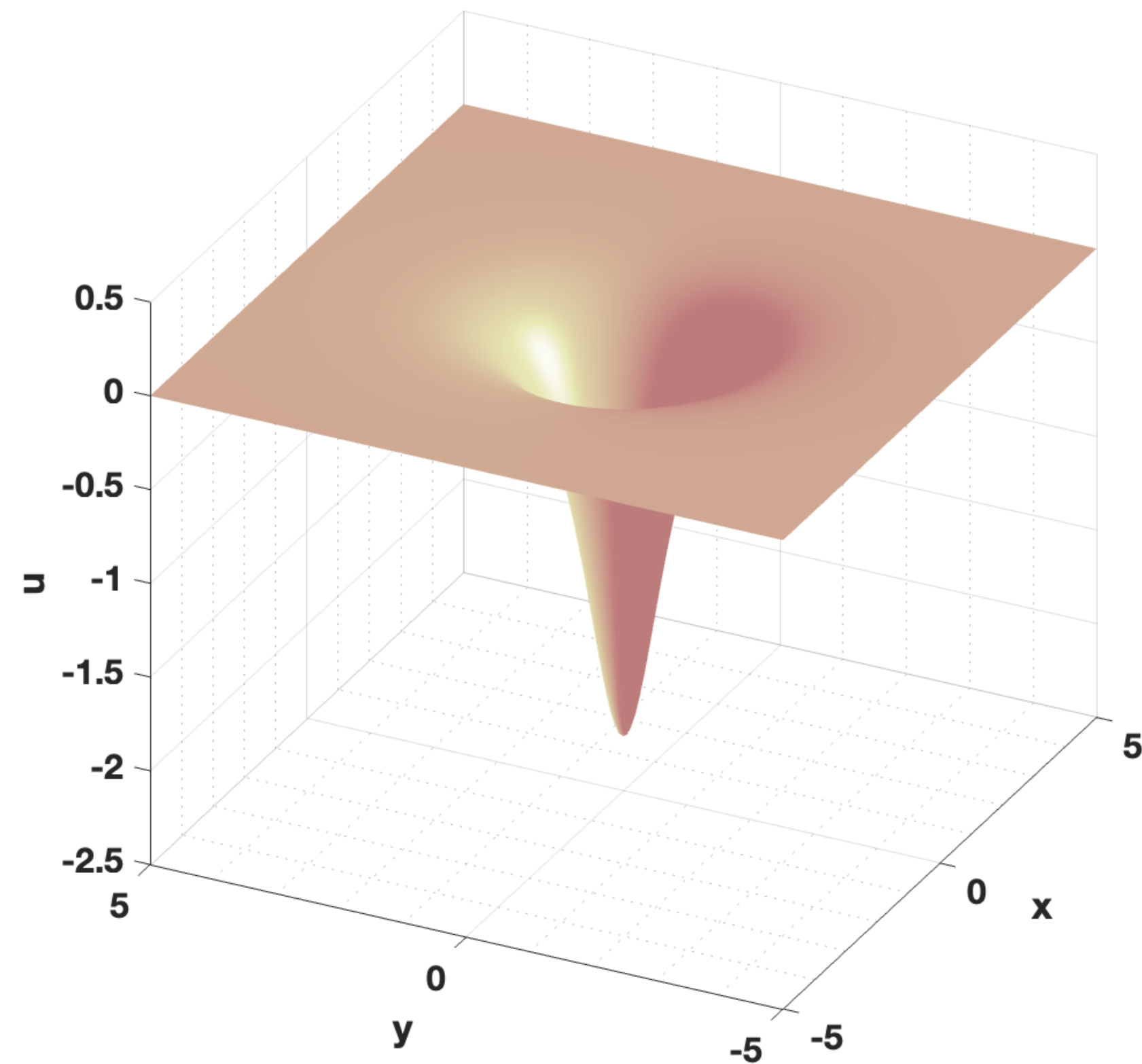


$$u(x, z) = \frac{8ia[2ax \sinh 2ax - \cosh 2ax - 4ia^2z \cosh 2ax]}{\cosh 4ax + 1 + 8a^2x^2 + 32a^4z^2} \exp(2ia^2z) \quad (\text{ISM})$$

# История солитонов

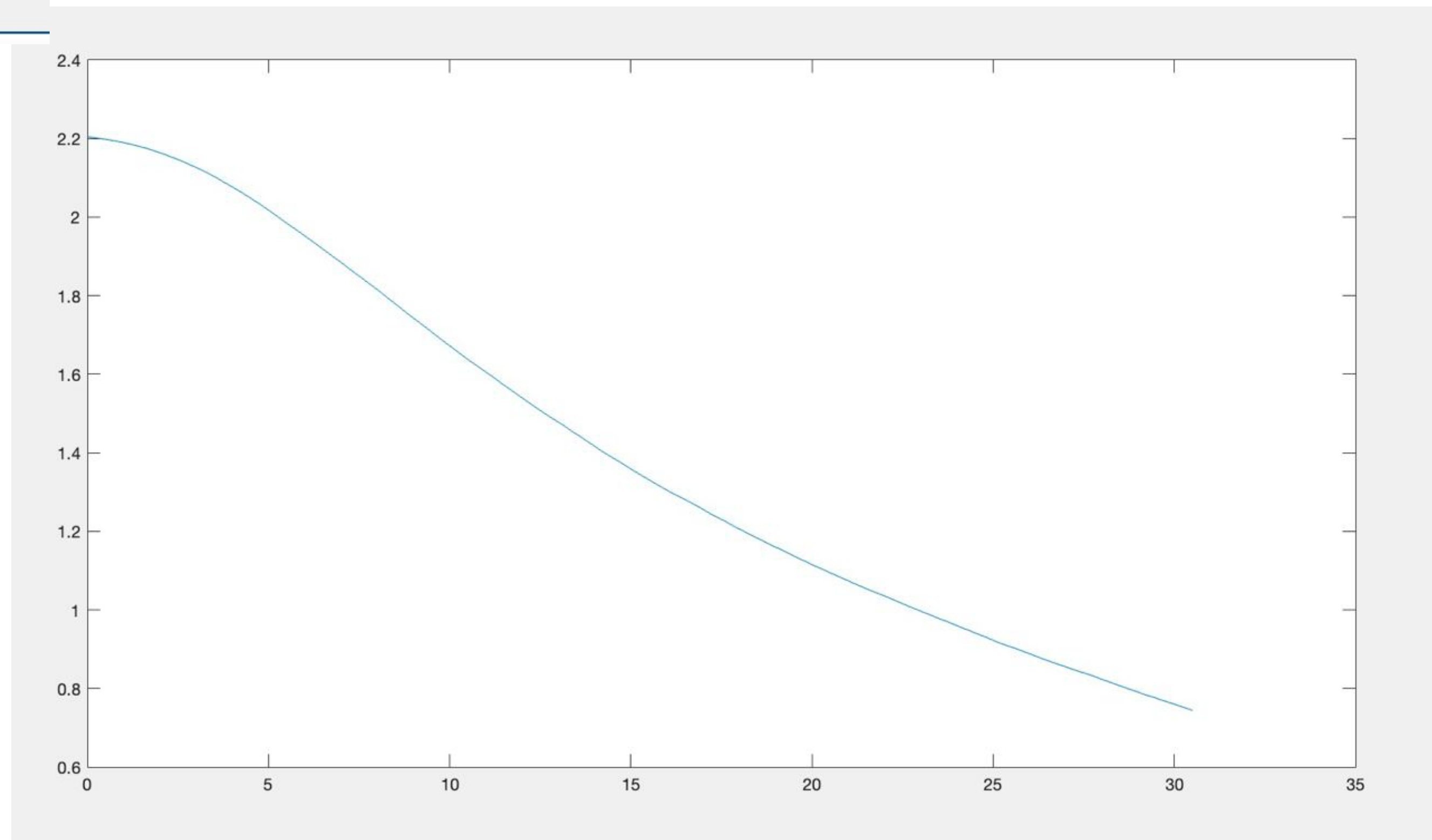
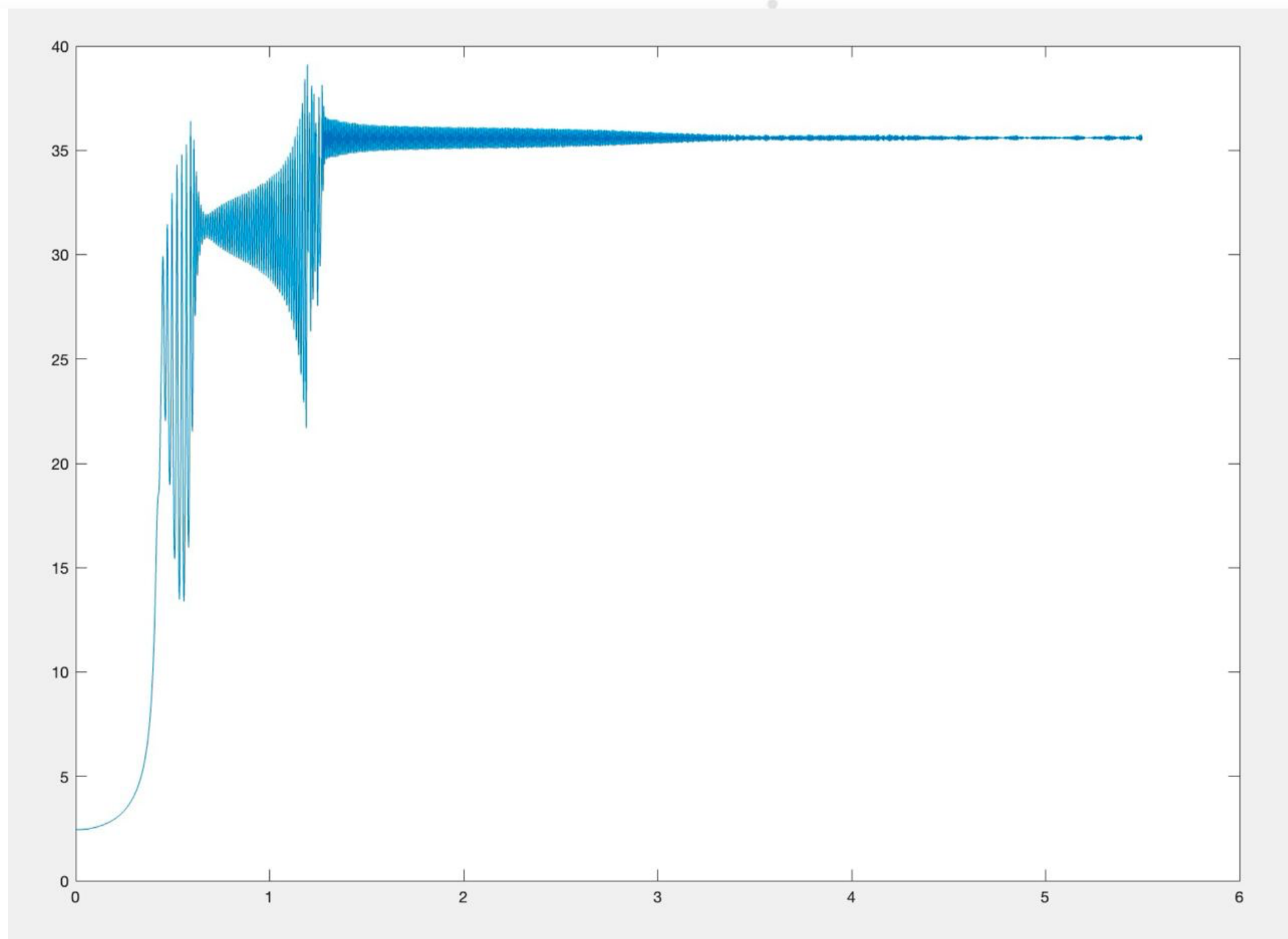
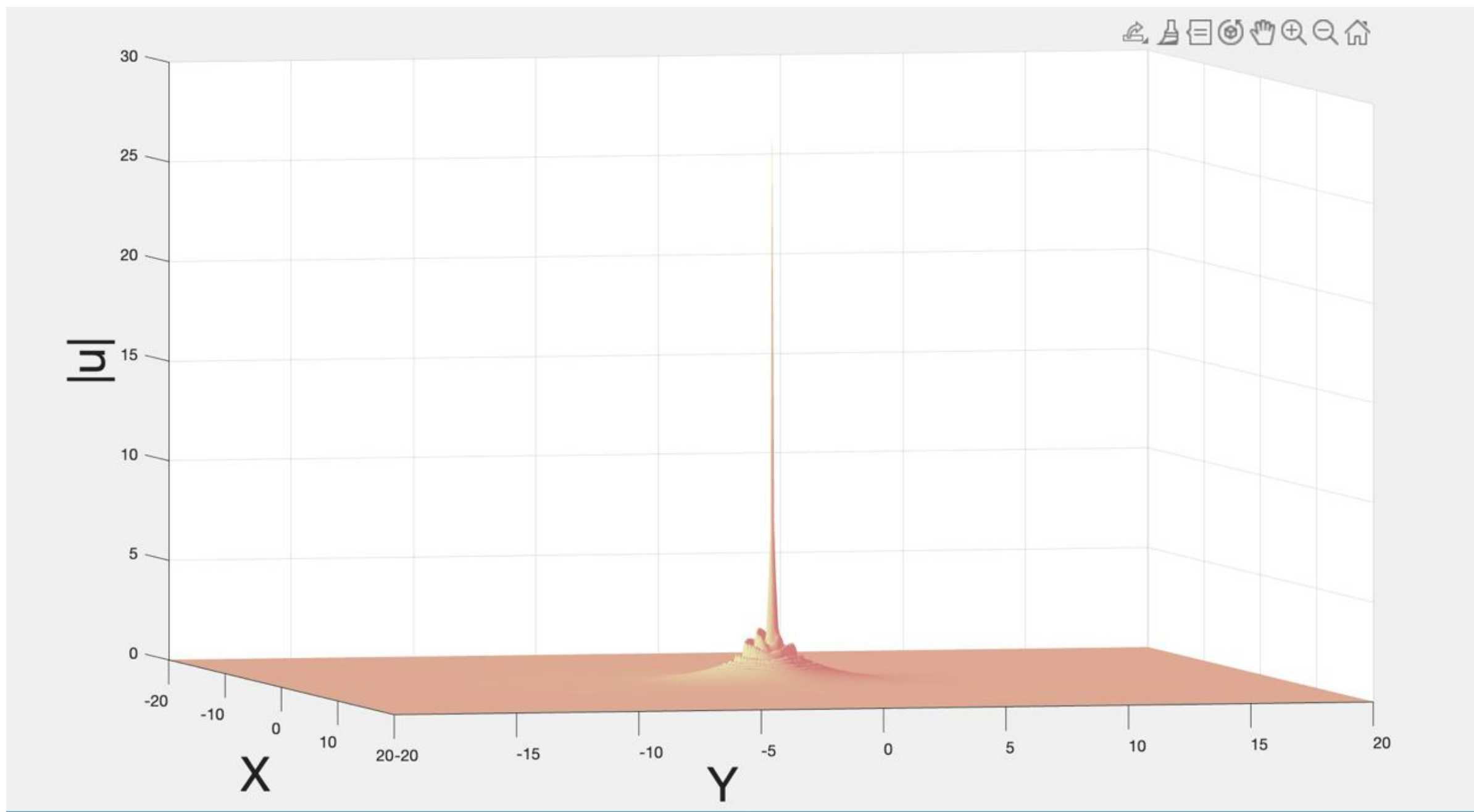


# Теперь в 2-d (Солитон Таунса)

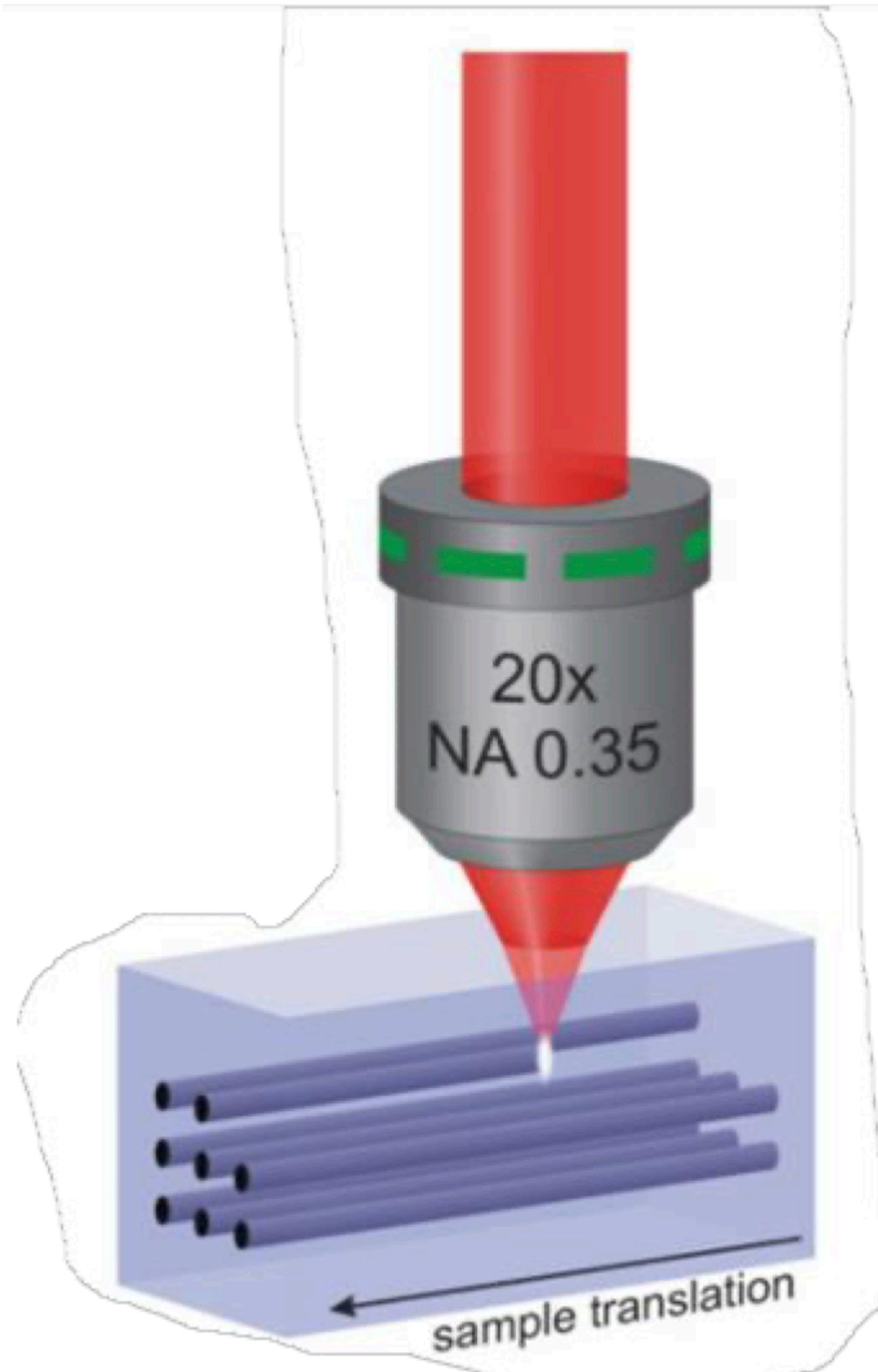


$$i \frac{\partial u}{\partial z} = -\frac{1}{2} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \sigma |u|^2 u$$

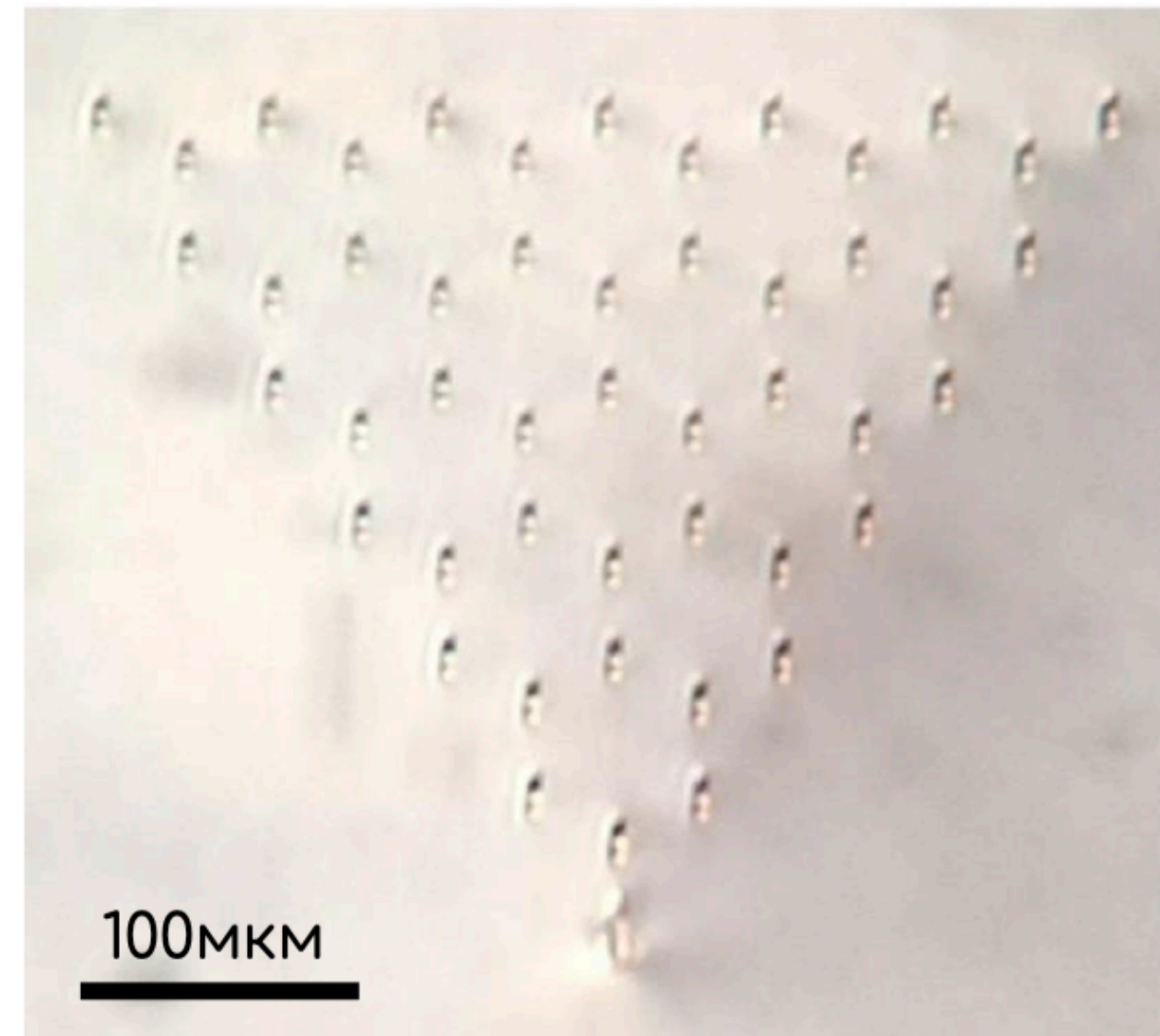
$$U = \int |\psi|^2 dx dy = 5.85$$



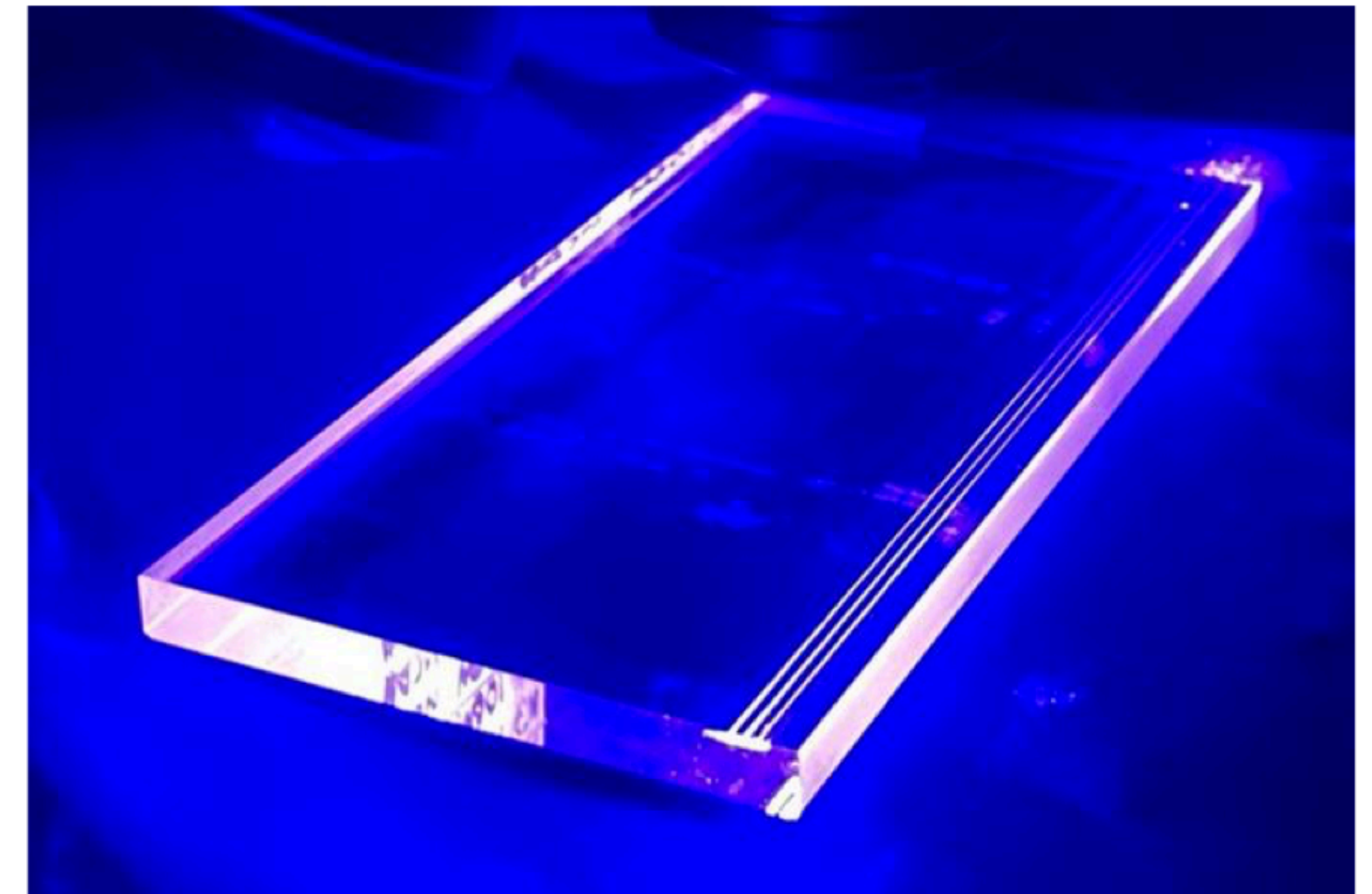
# Изготовление образцов



Szameit, A, and S. Nolte , "Discrete optics in femtosecond-laser written photonic structures," *J. Phys. B* 43 (16), 163001 (2010)

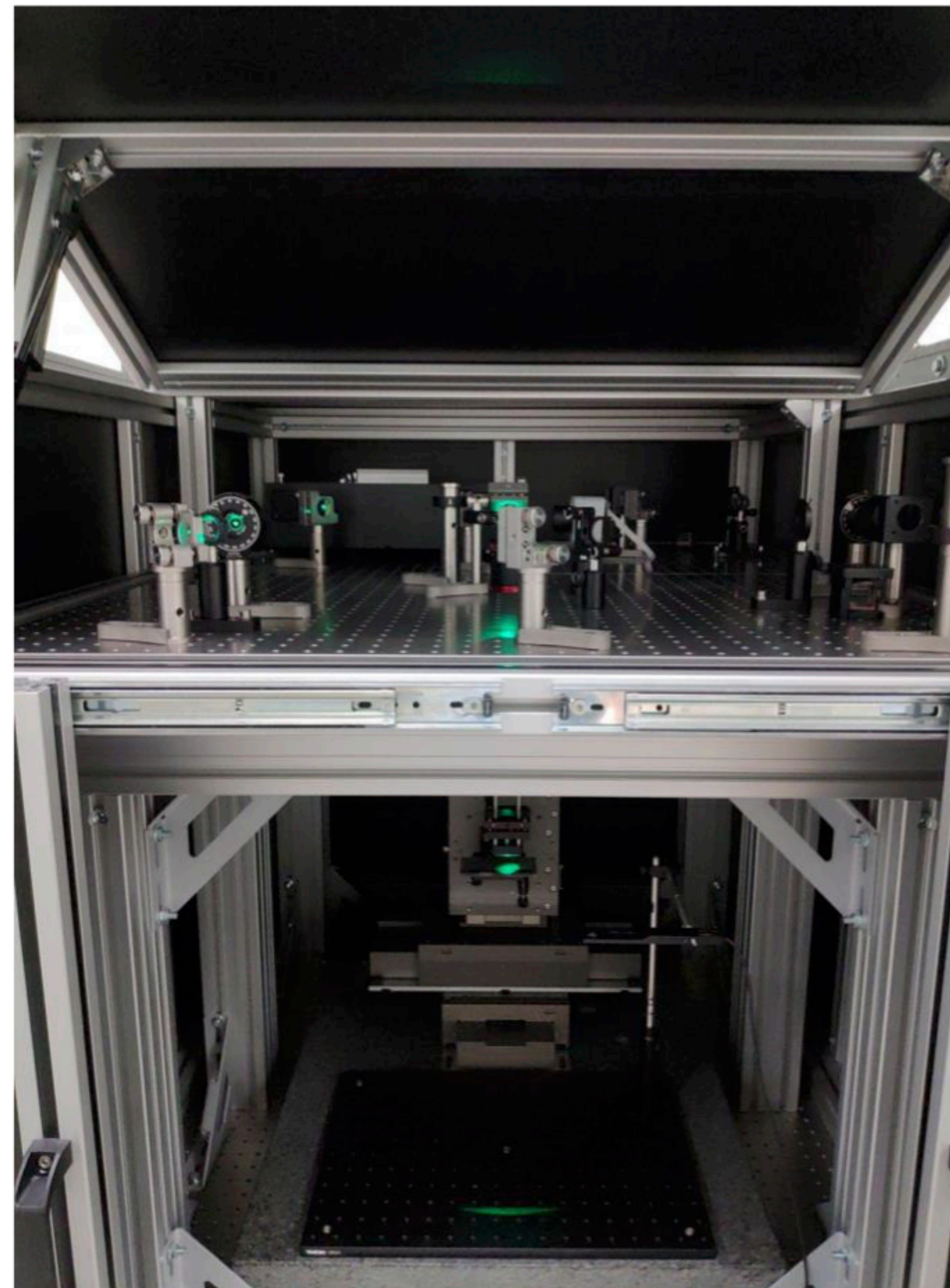


*Микрофотография торца массива волноводов*



*Фотография 10см образца с тремя волноводными массивами*

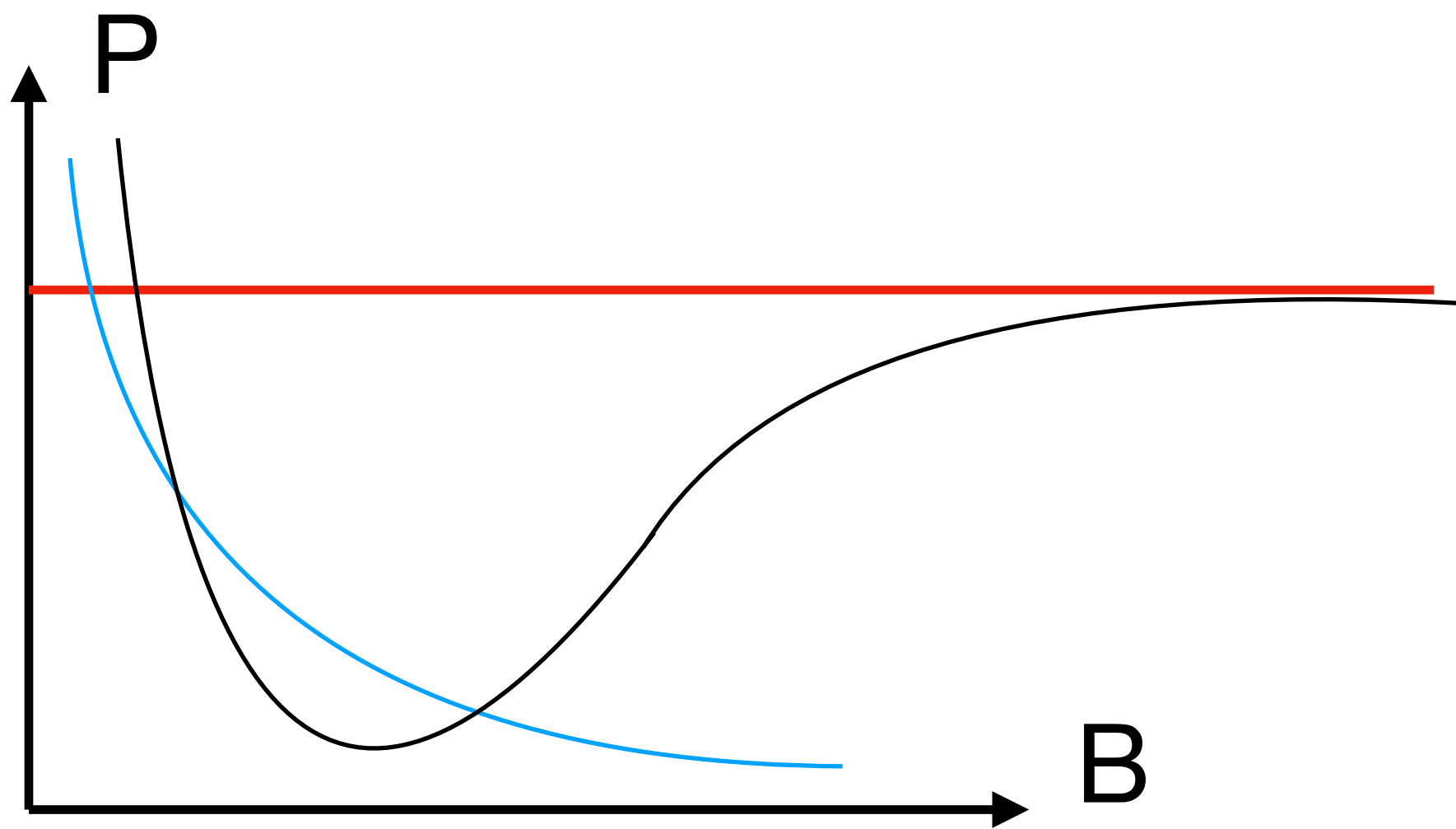
# Лазерная запись в ЦКТ МГУ



# Критерий Вахитова-Колоколова

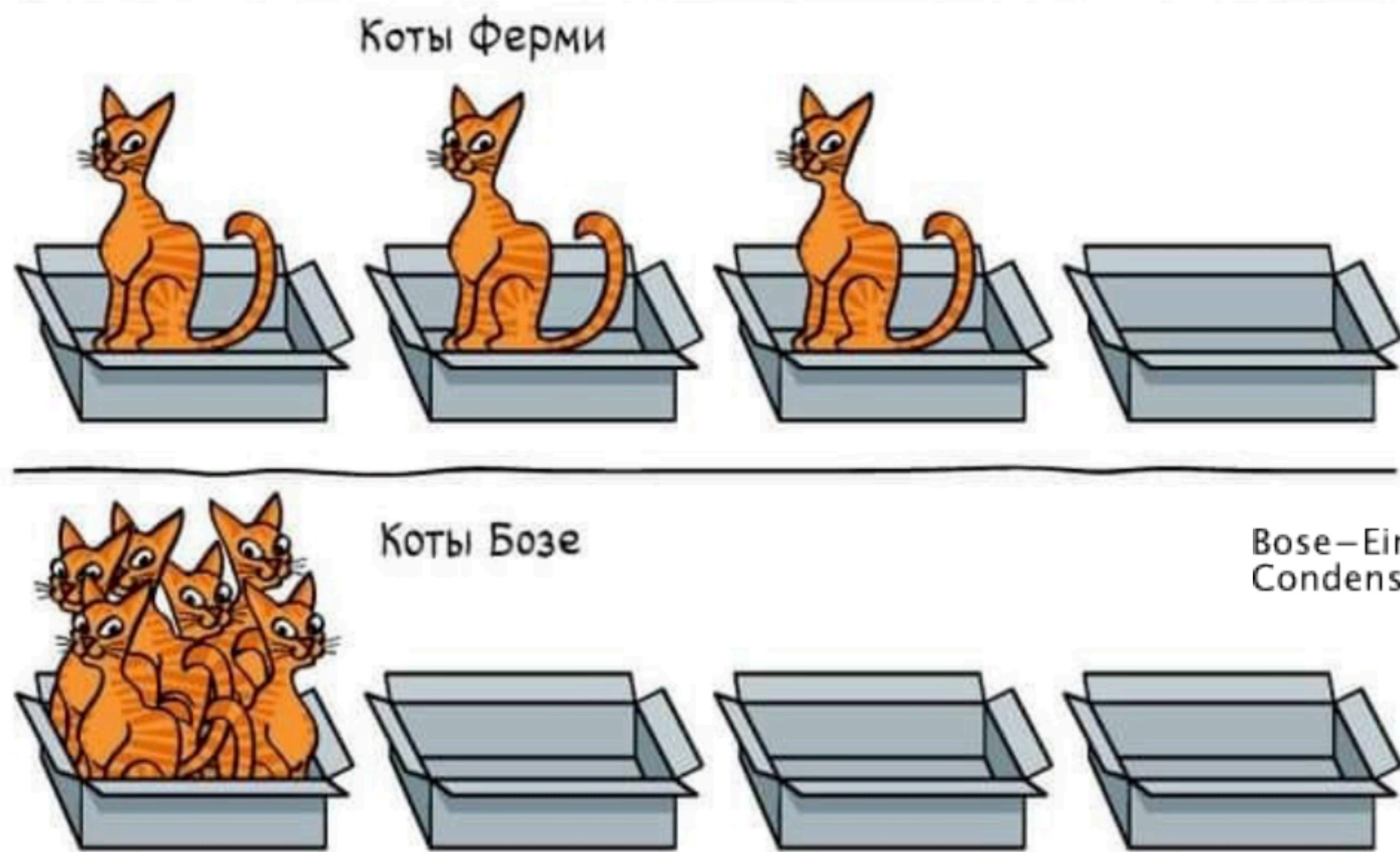
$$i \frac{\partial u}{\partial z} = -\frac{1}{2} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \sigma |u|^2 u + R(x, y)u$$

$$\frac{\partial P}{\partial \beta} > 0$$

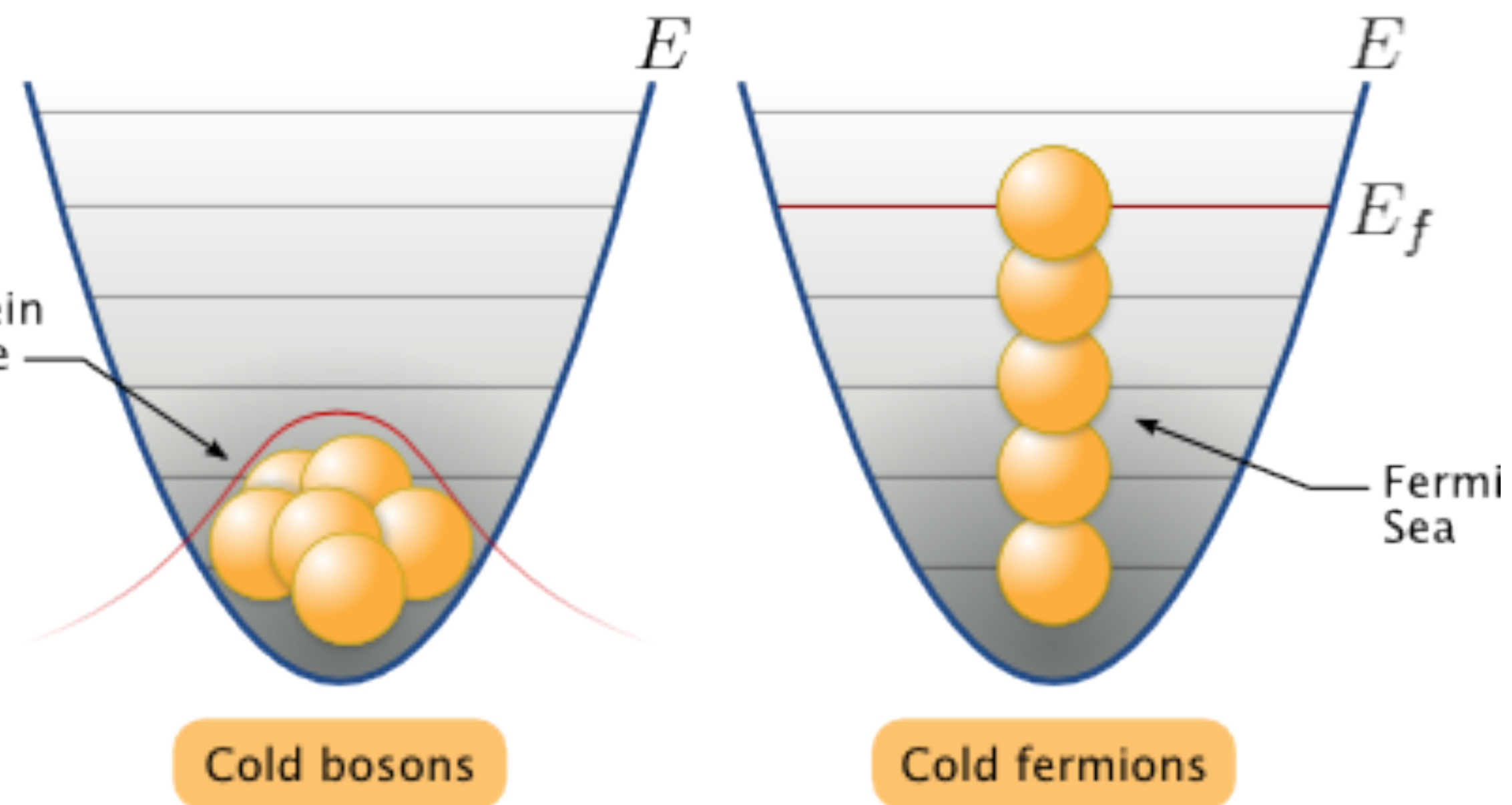




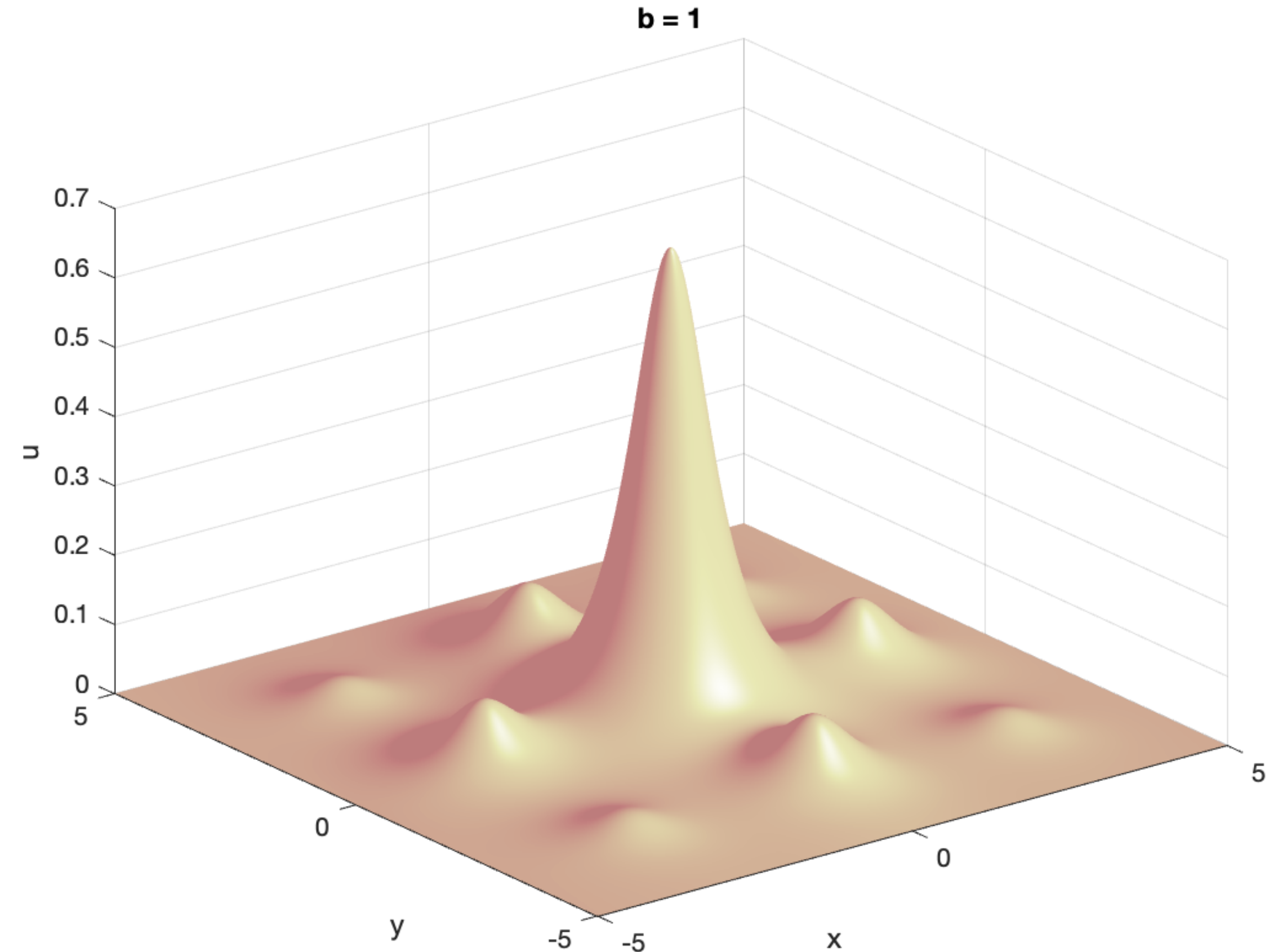
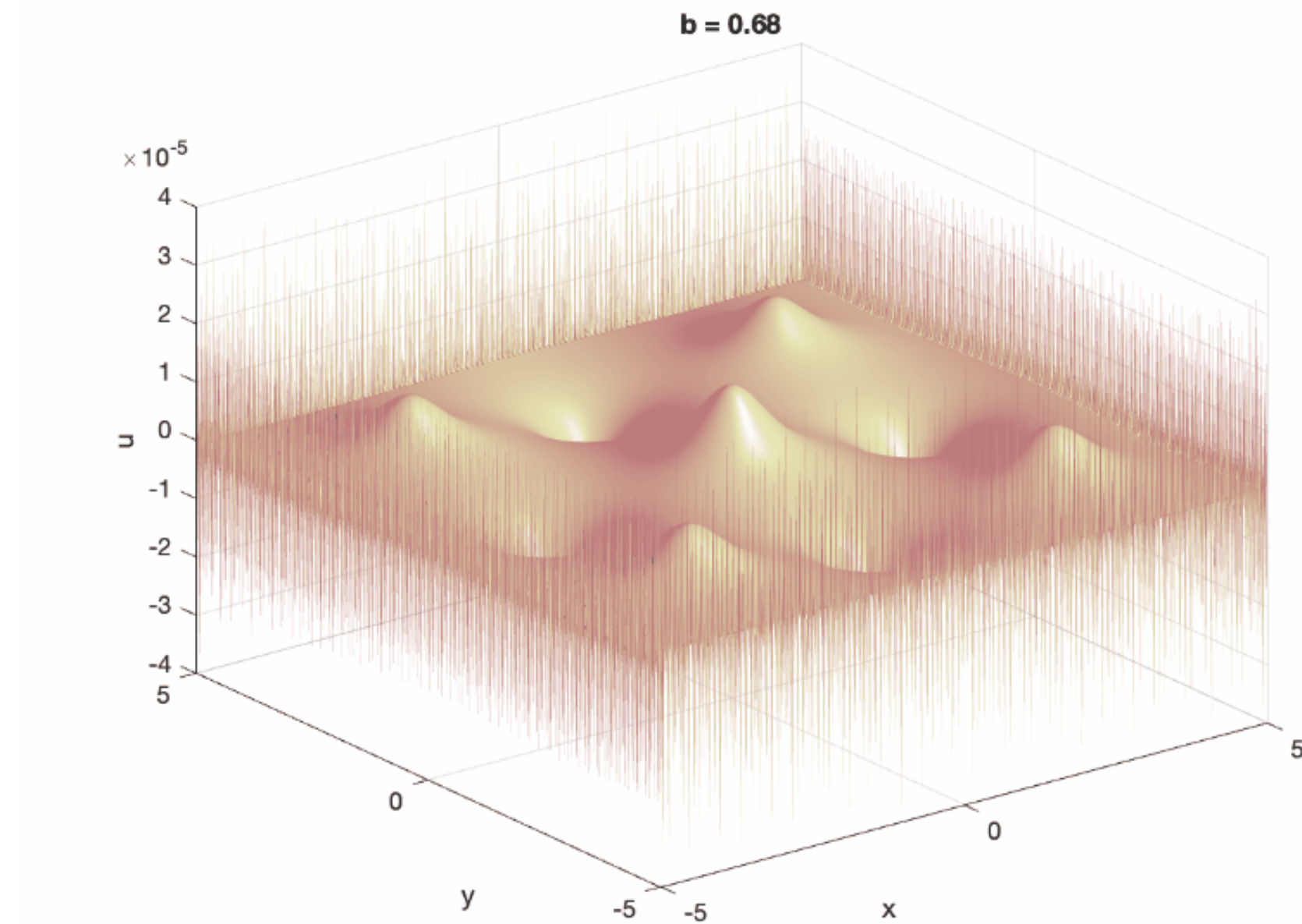
# Бозоны и фермионы



Bose–Einstein  
Condensate



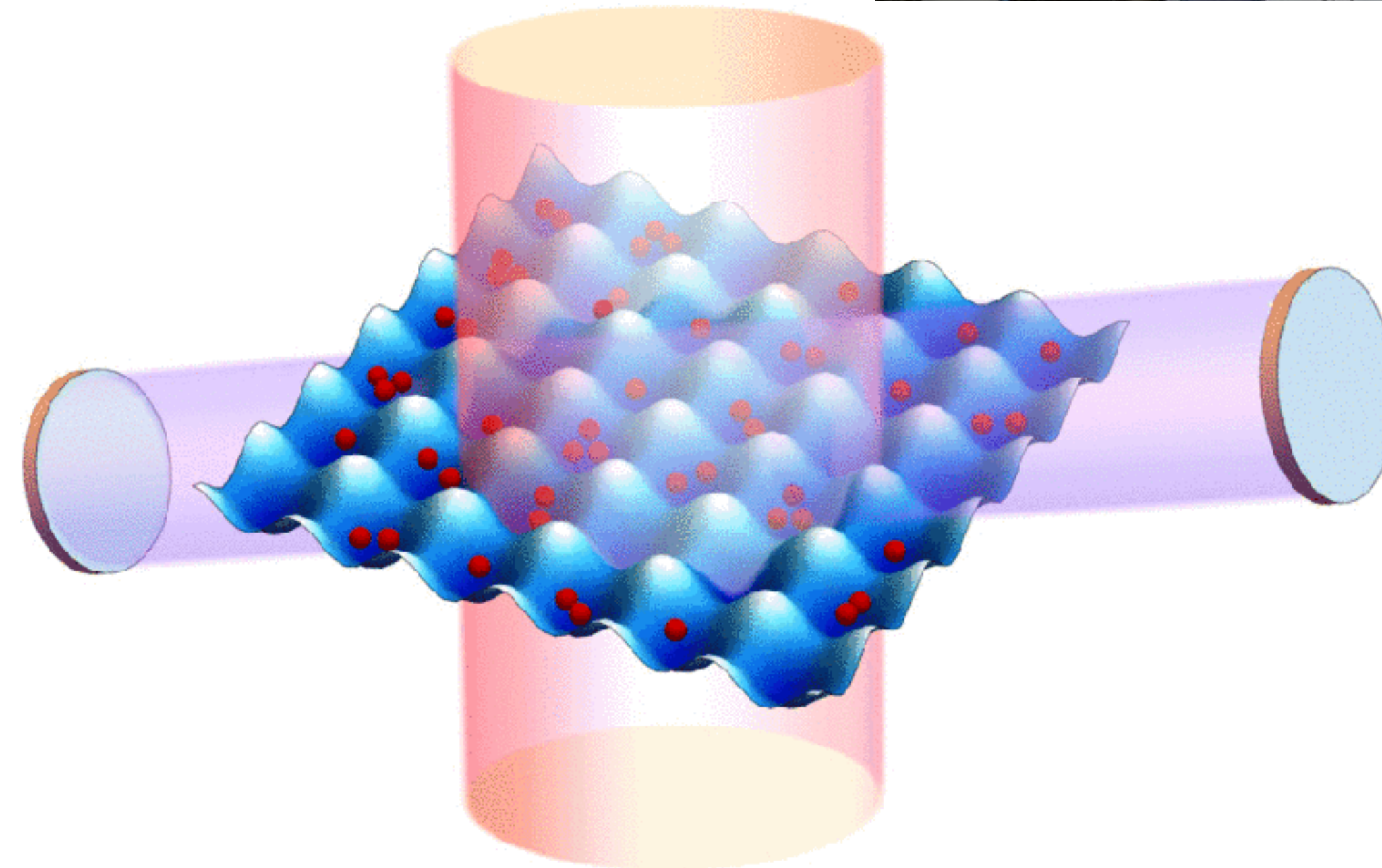
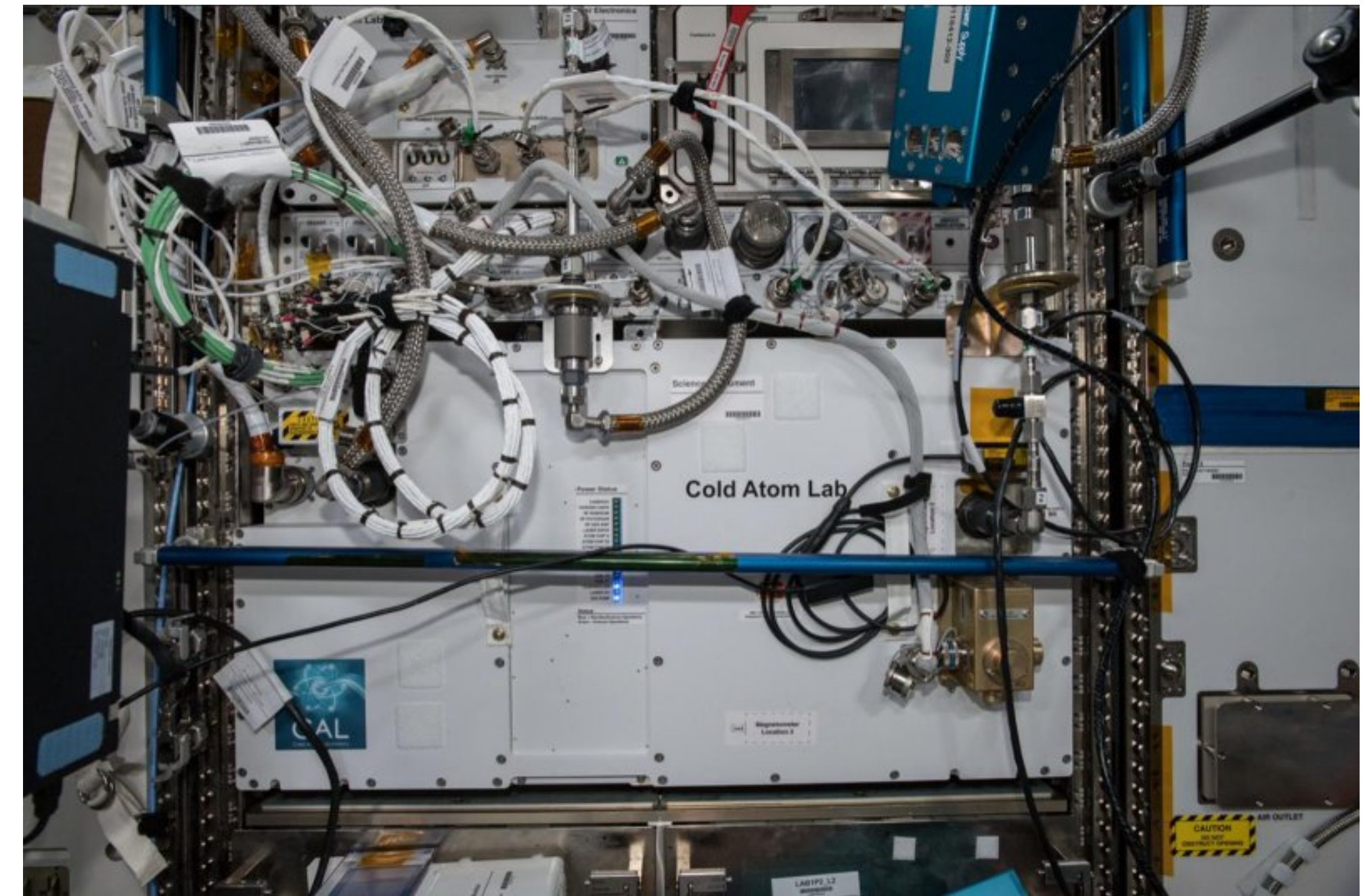
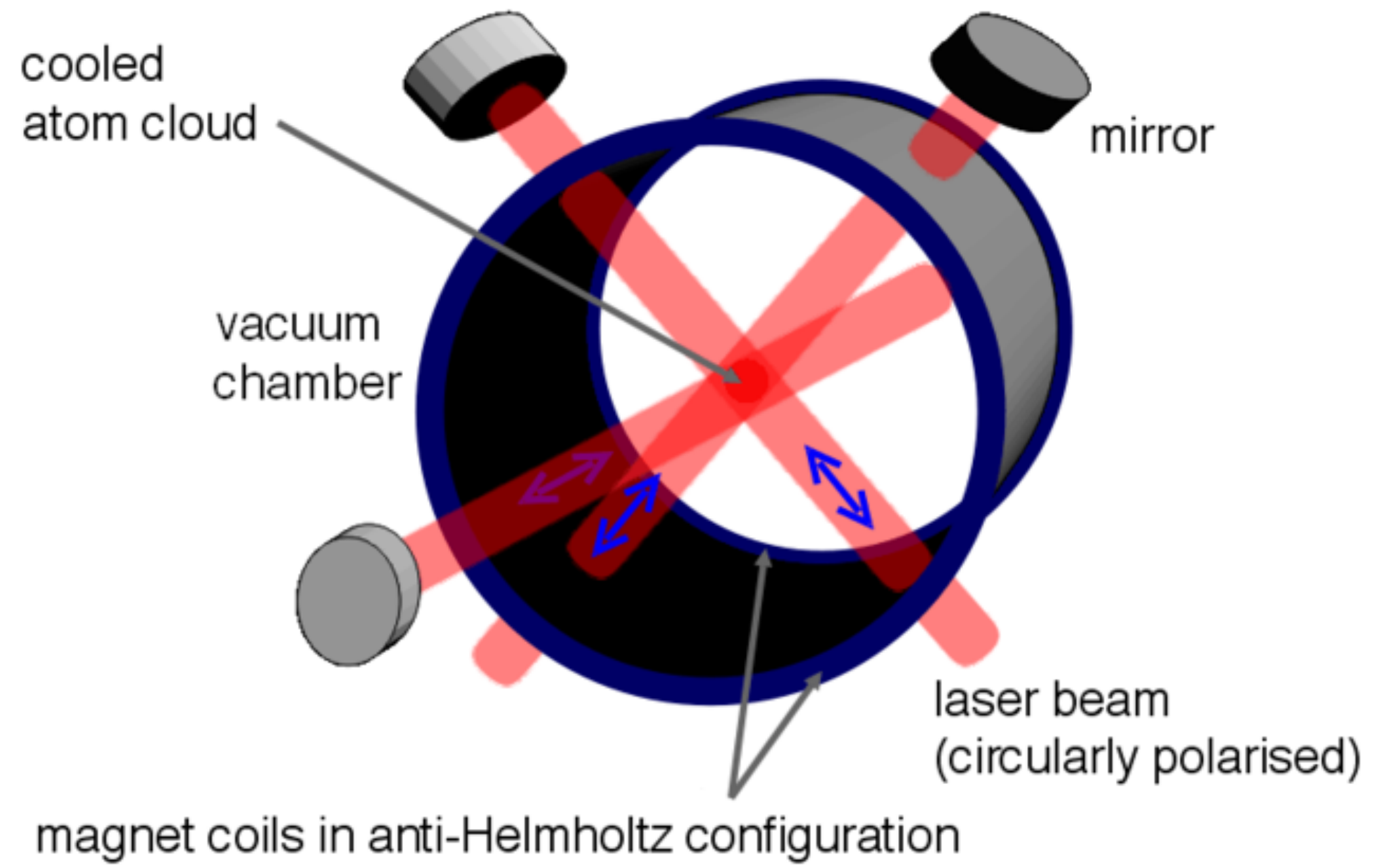
# BEC(bose-einstein condensate)



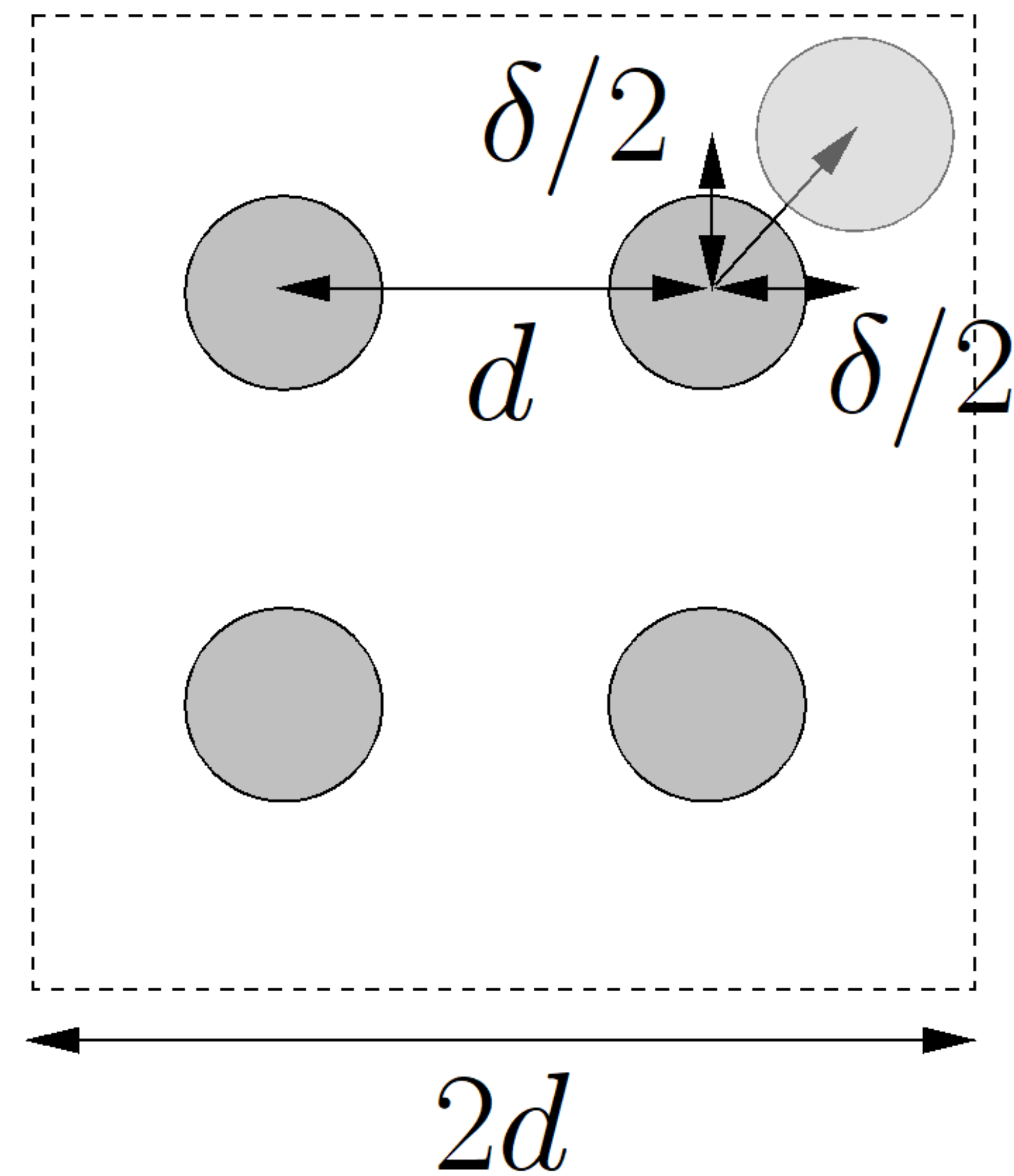
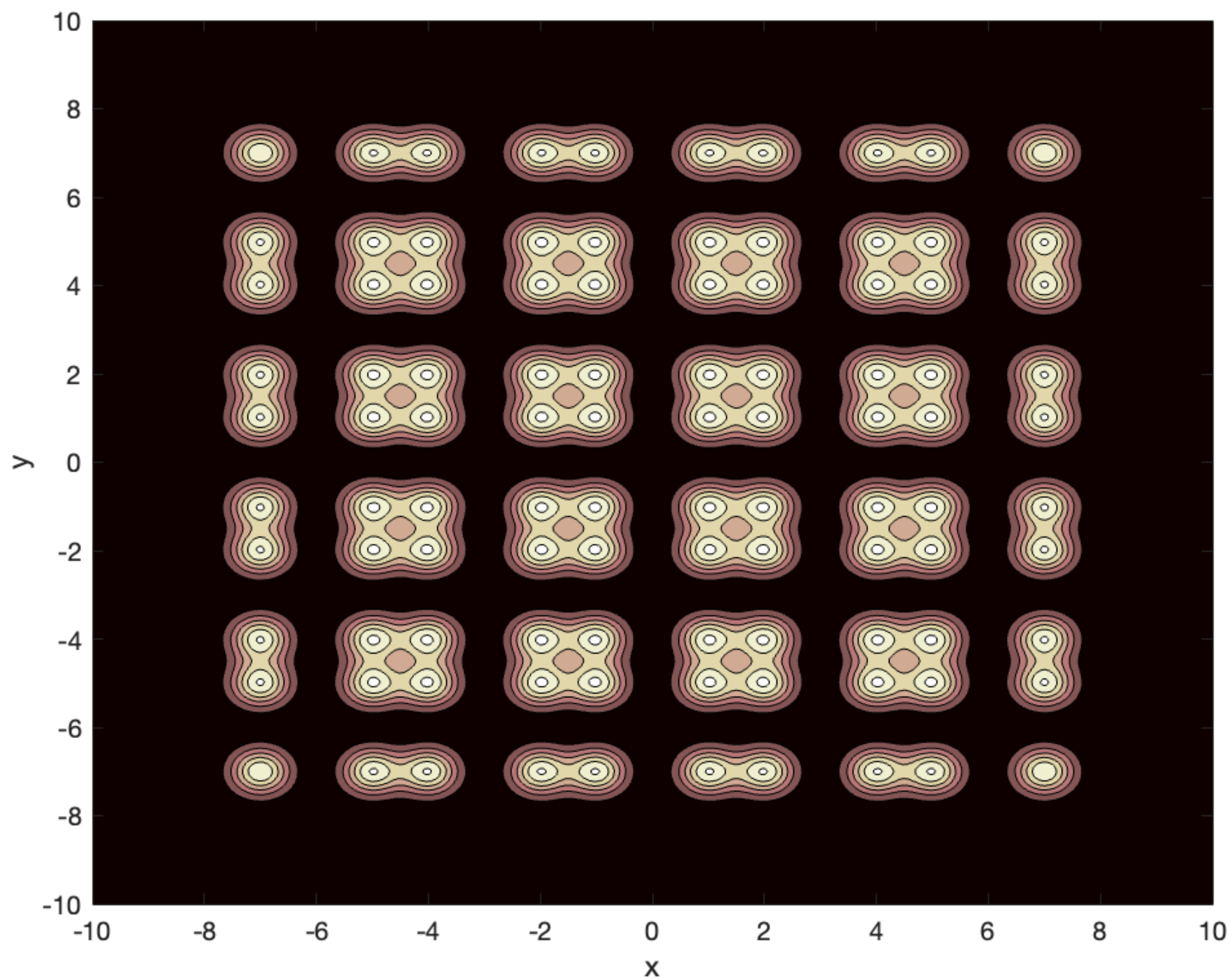
$$i \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(r) + \kappa |\psi|^2 \right) \psi$$

Теперь это уравнение Гросс-Питаевского :) 

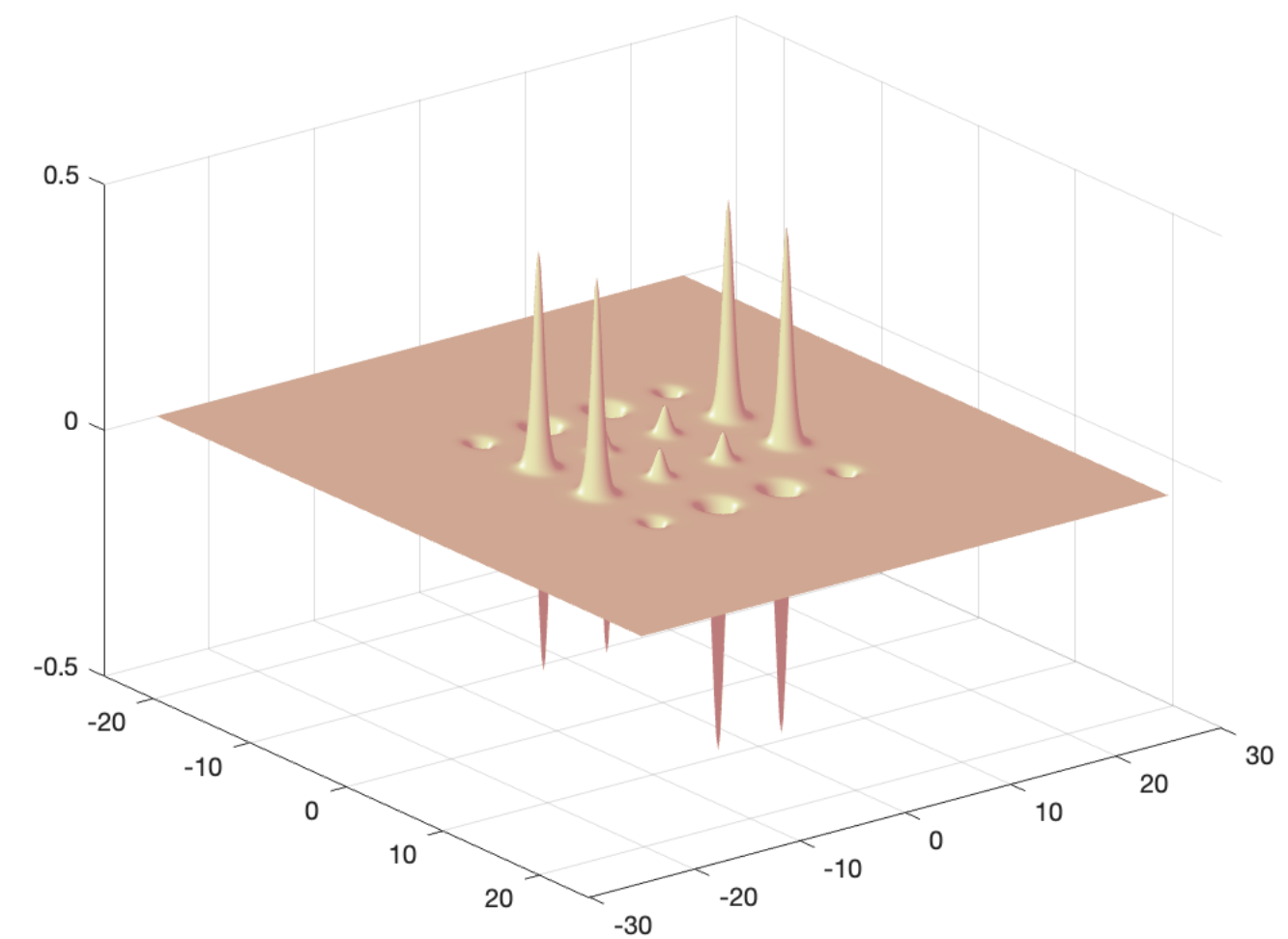
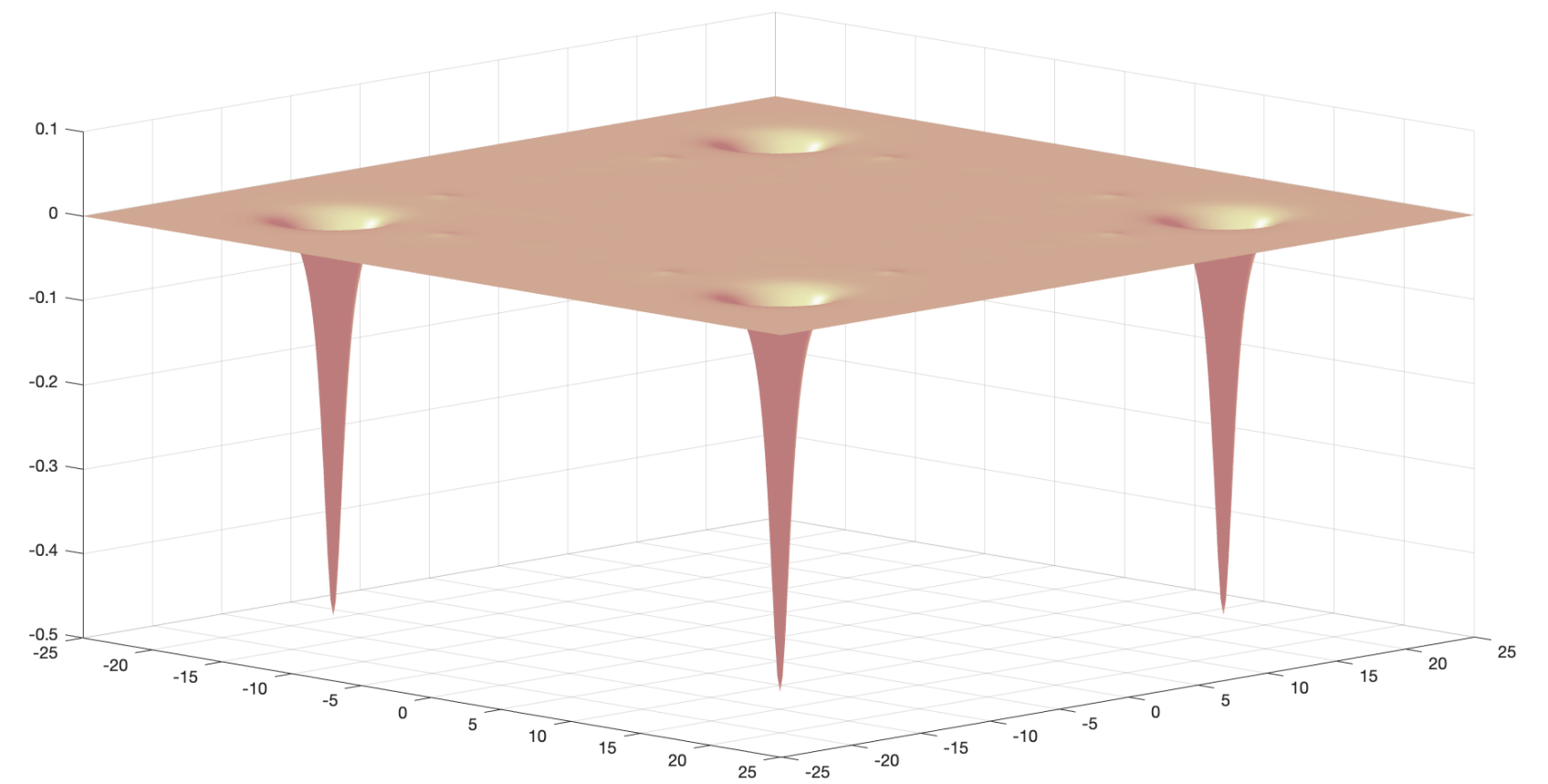
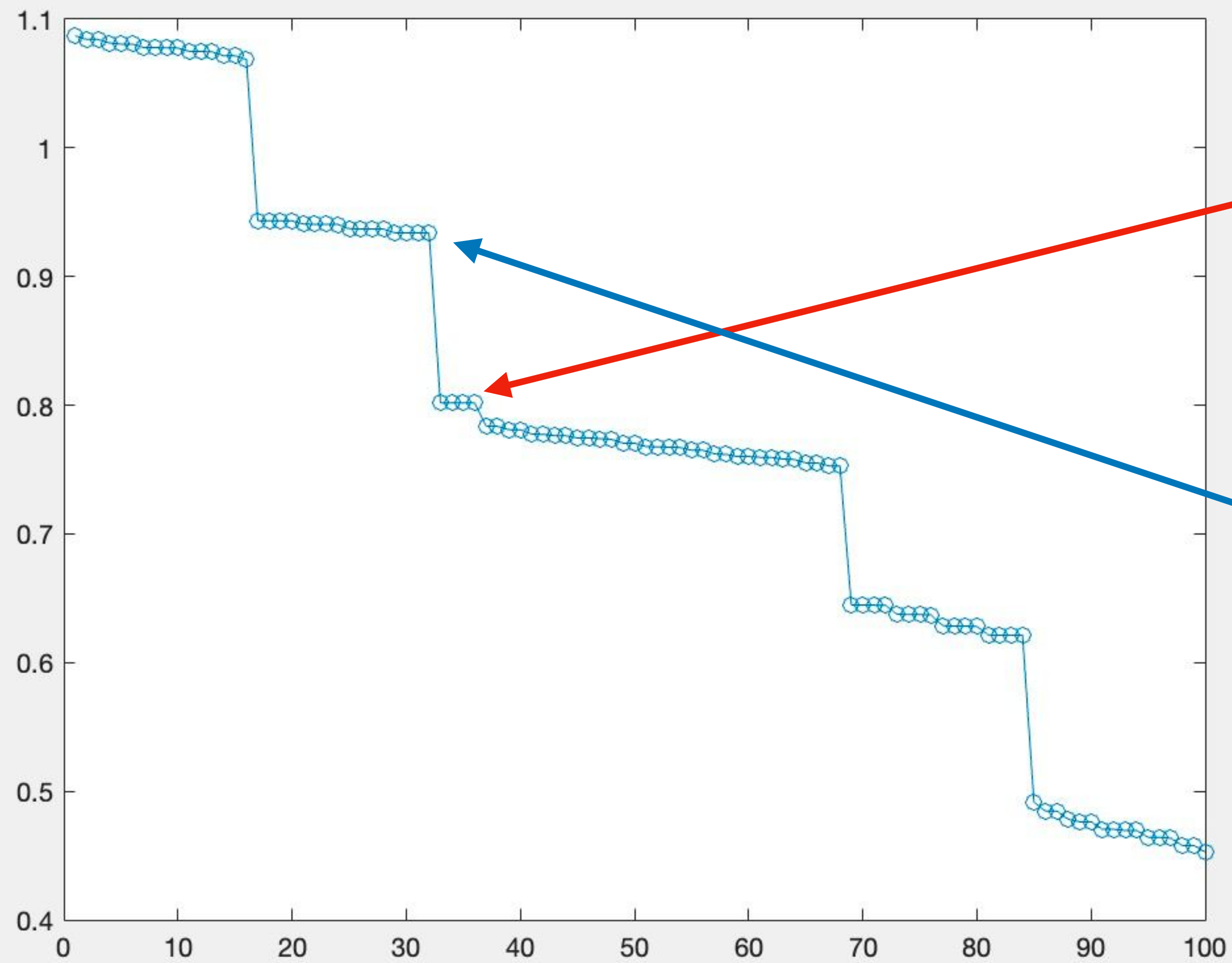
$$N = \int d\vec{r} |\psi(\vec{r})|^2$$



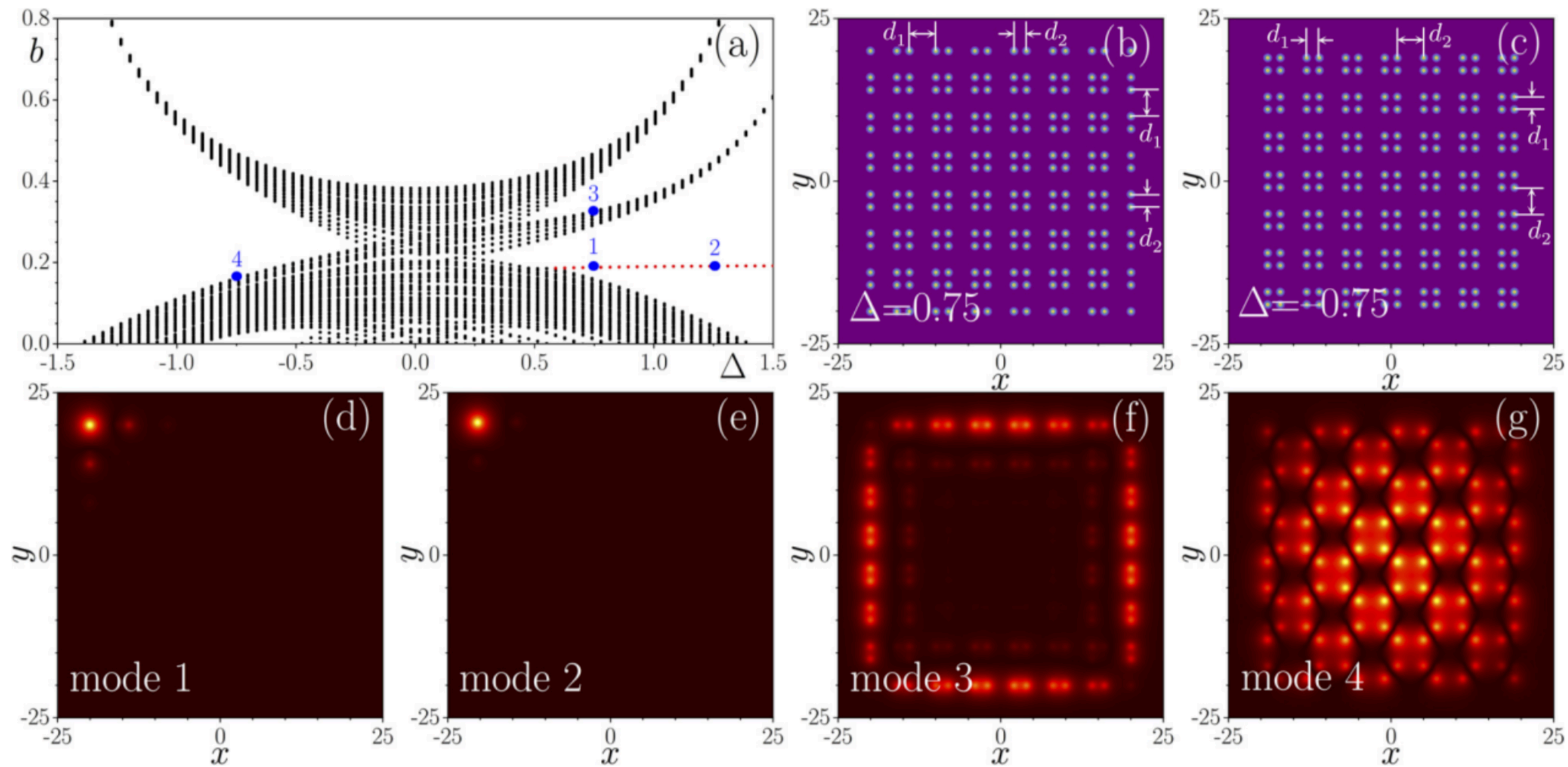
# 2-d модель Су-Шриффера-Хигера

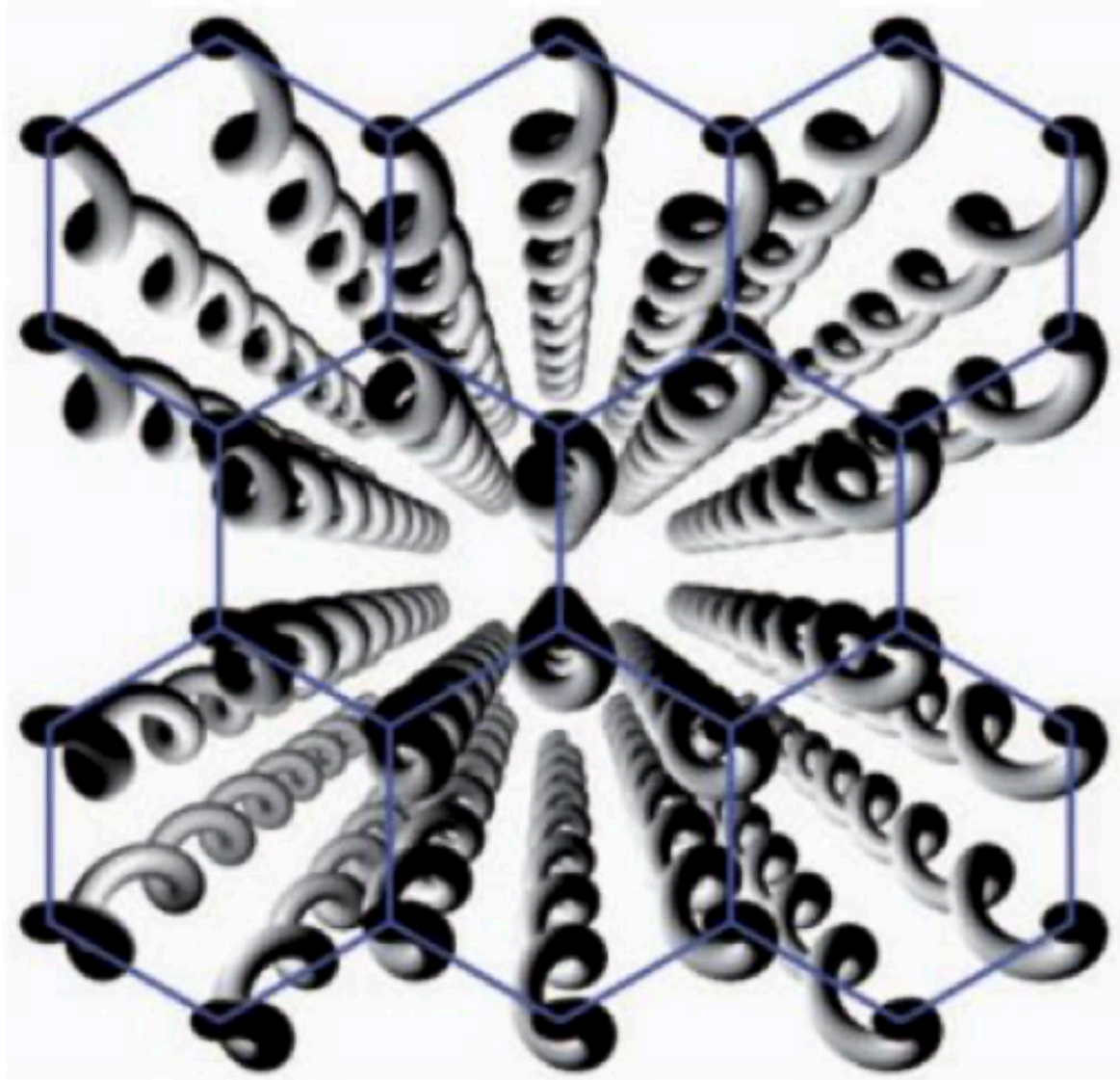


# Собственные значения

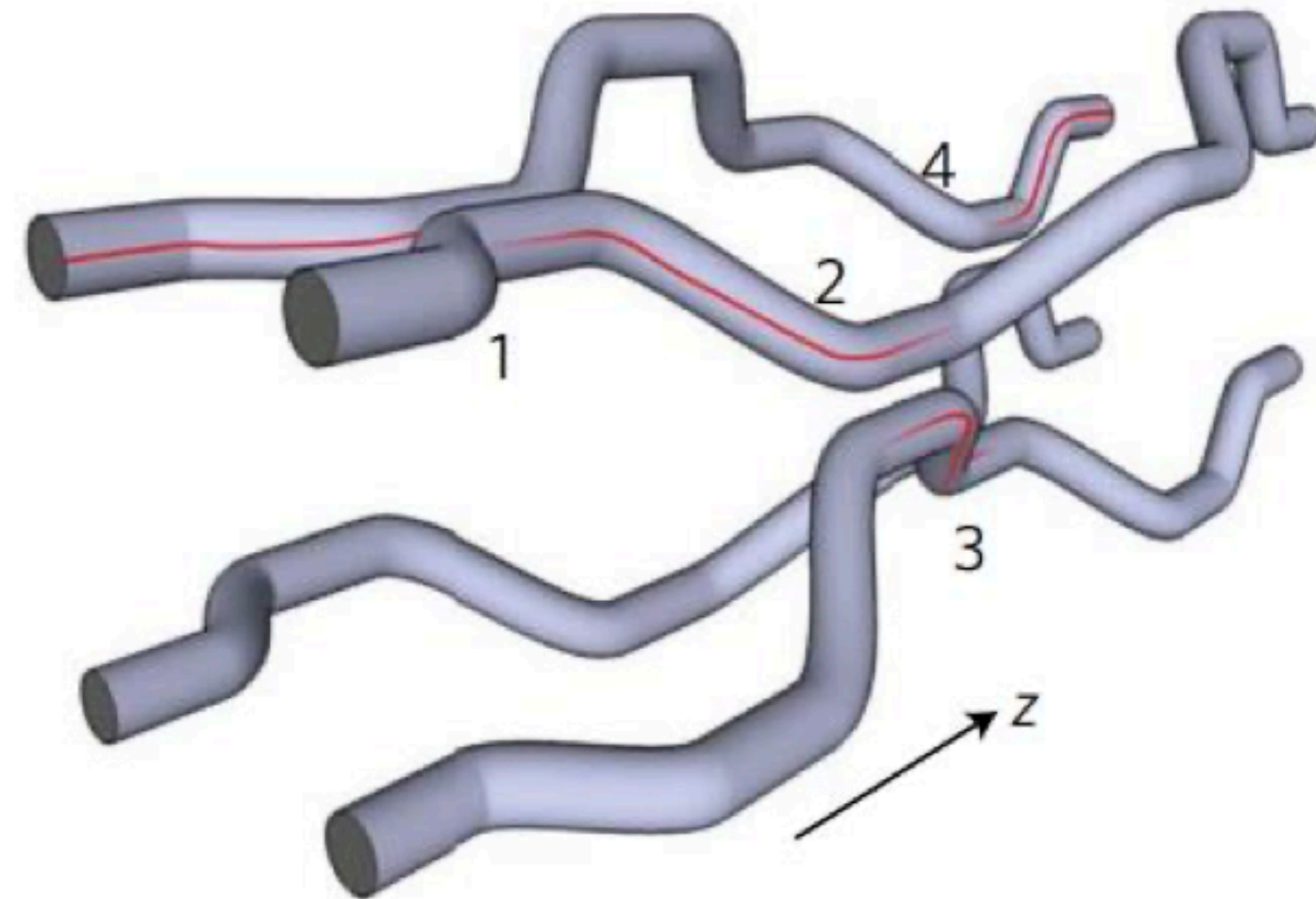


# Моды!

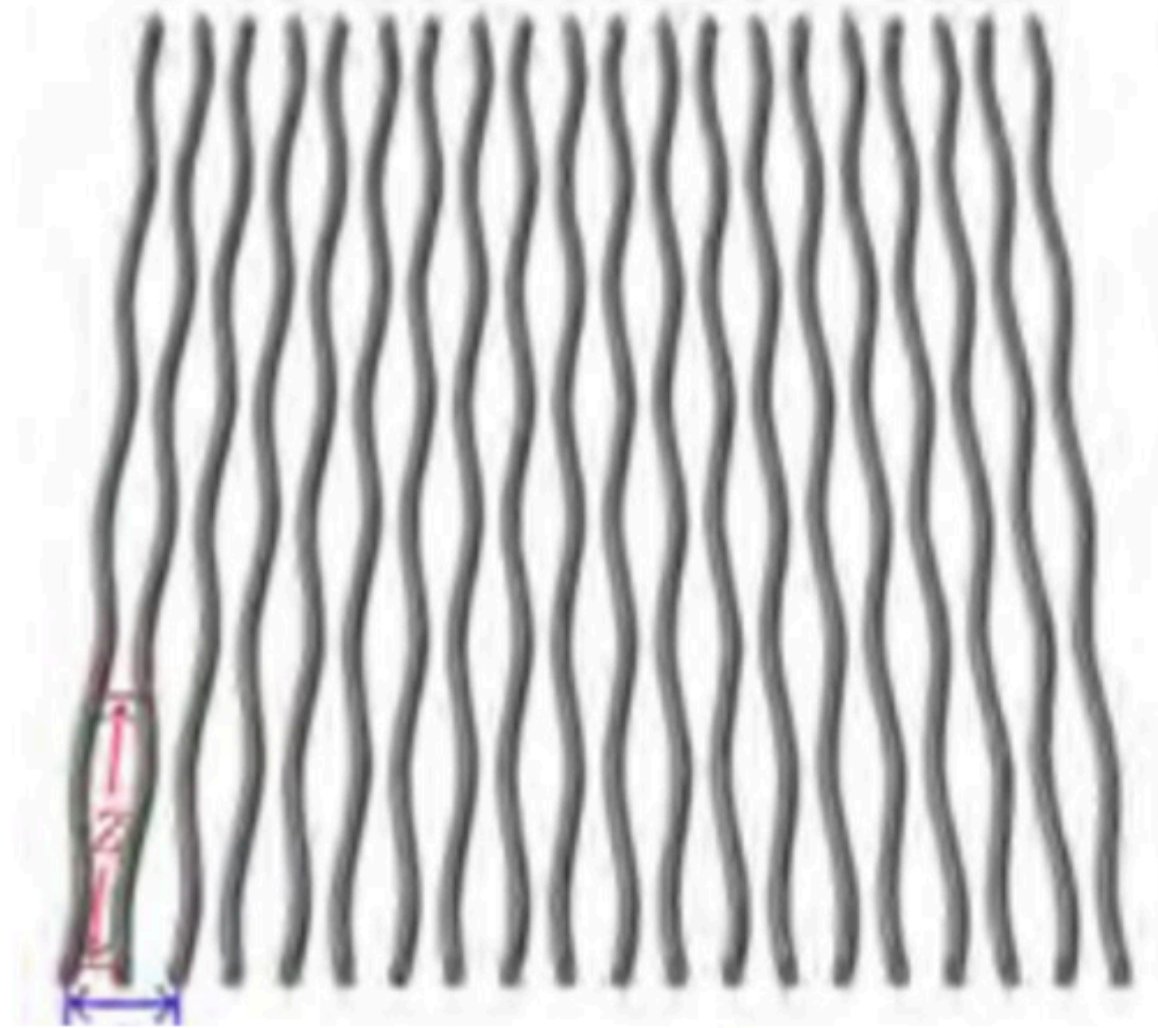




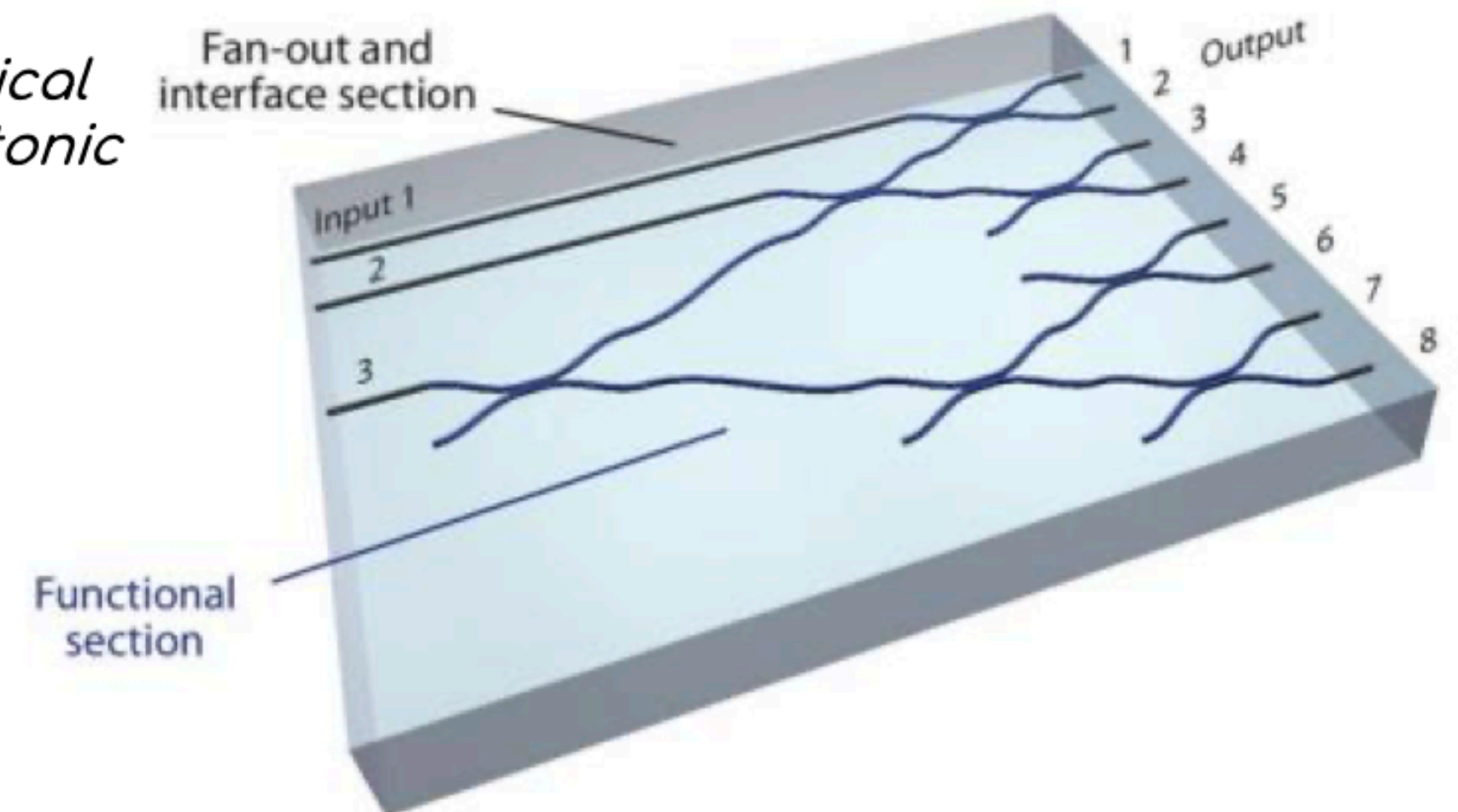
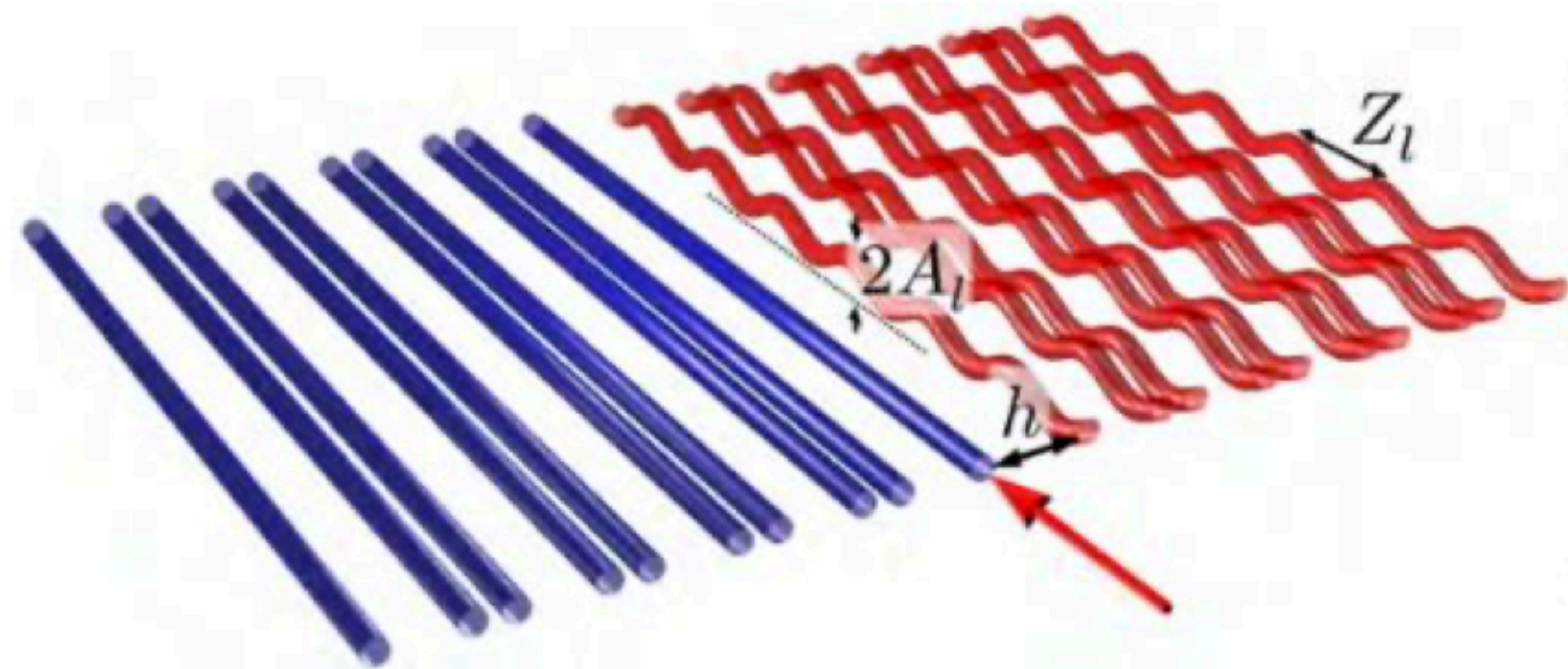
Rechtsman, M. C. et al. Photonic Floquet topological insulators. *Nature* 496, 196–200 (2013)



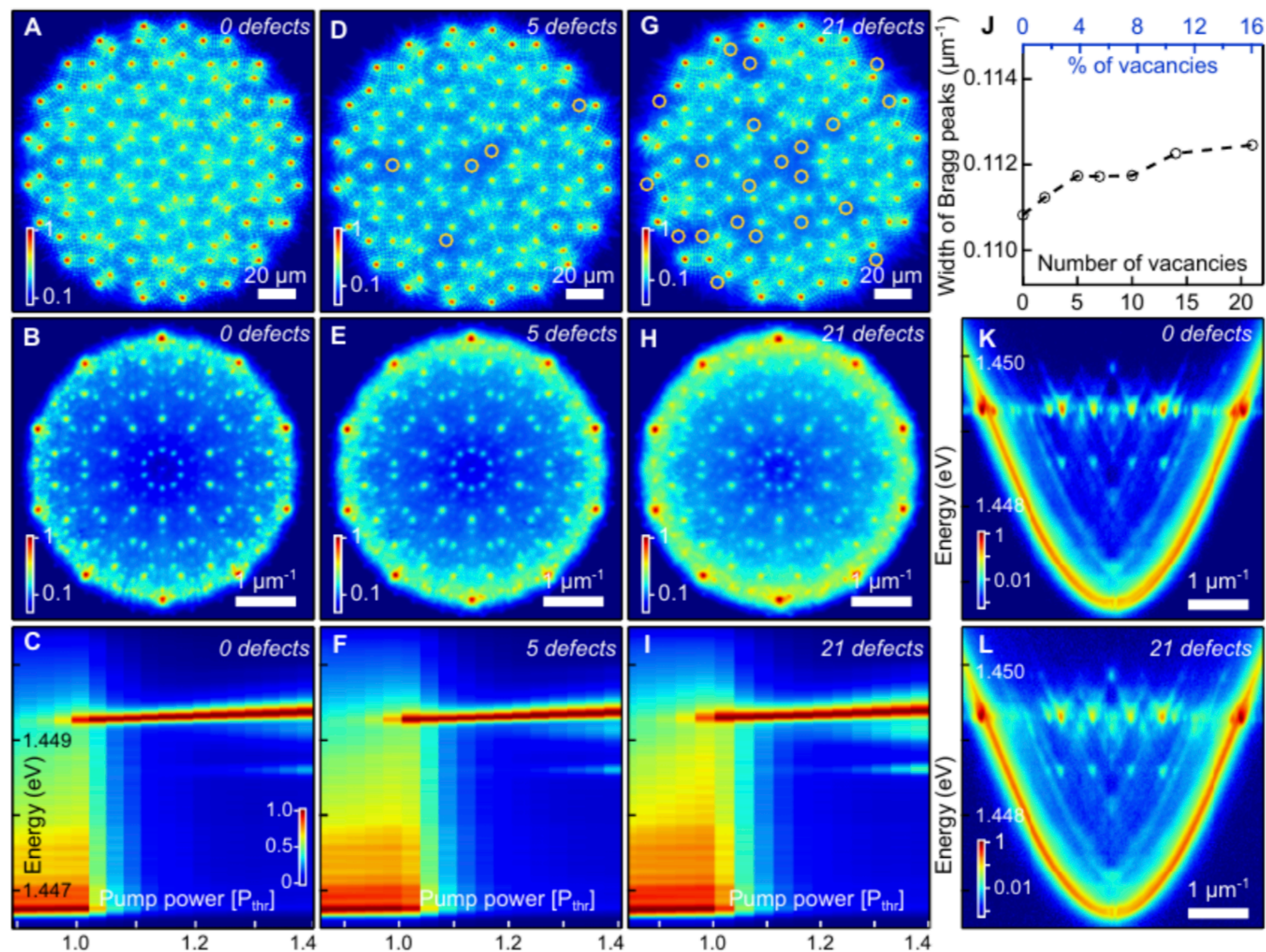
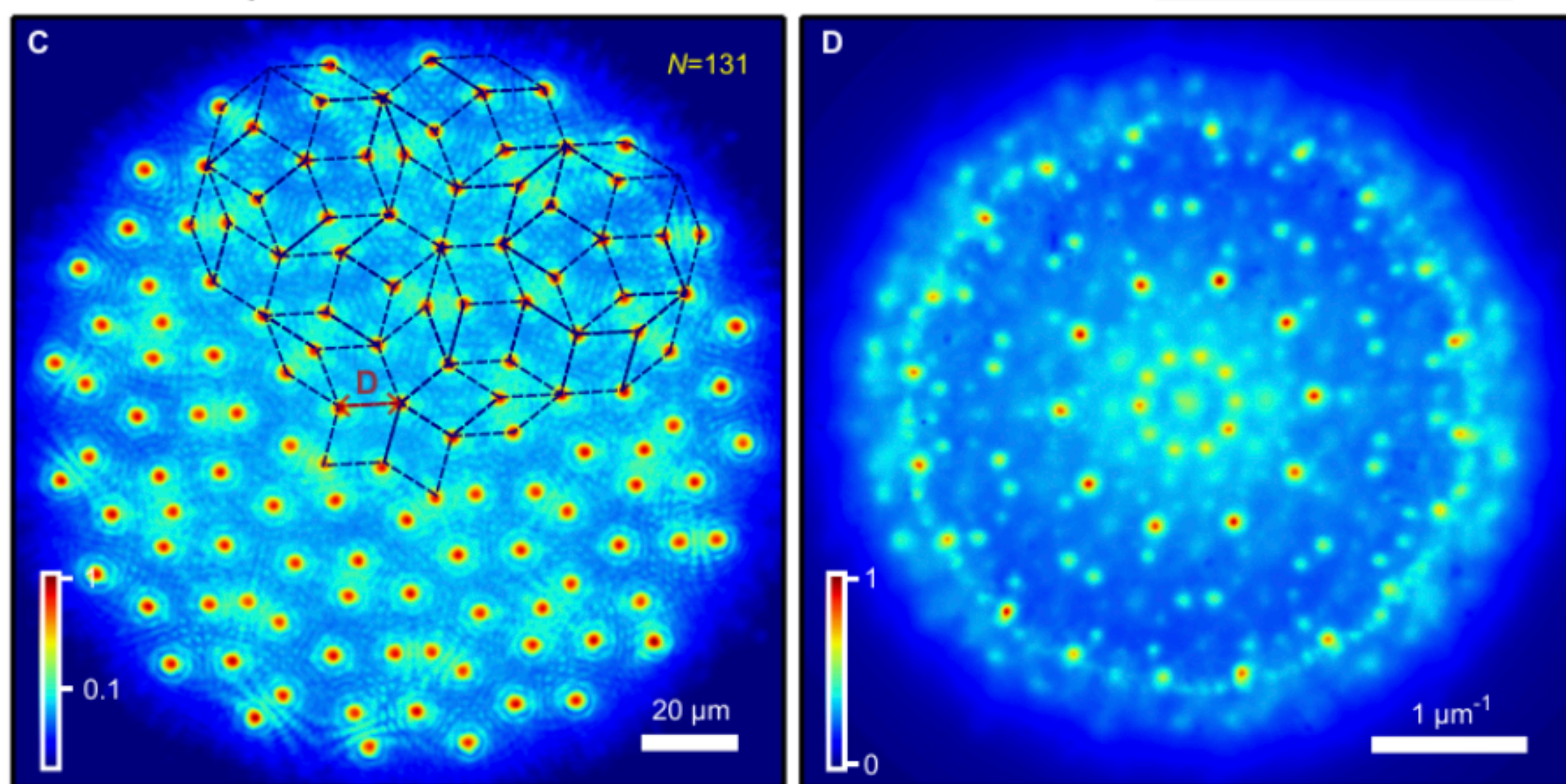
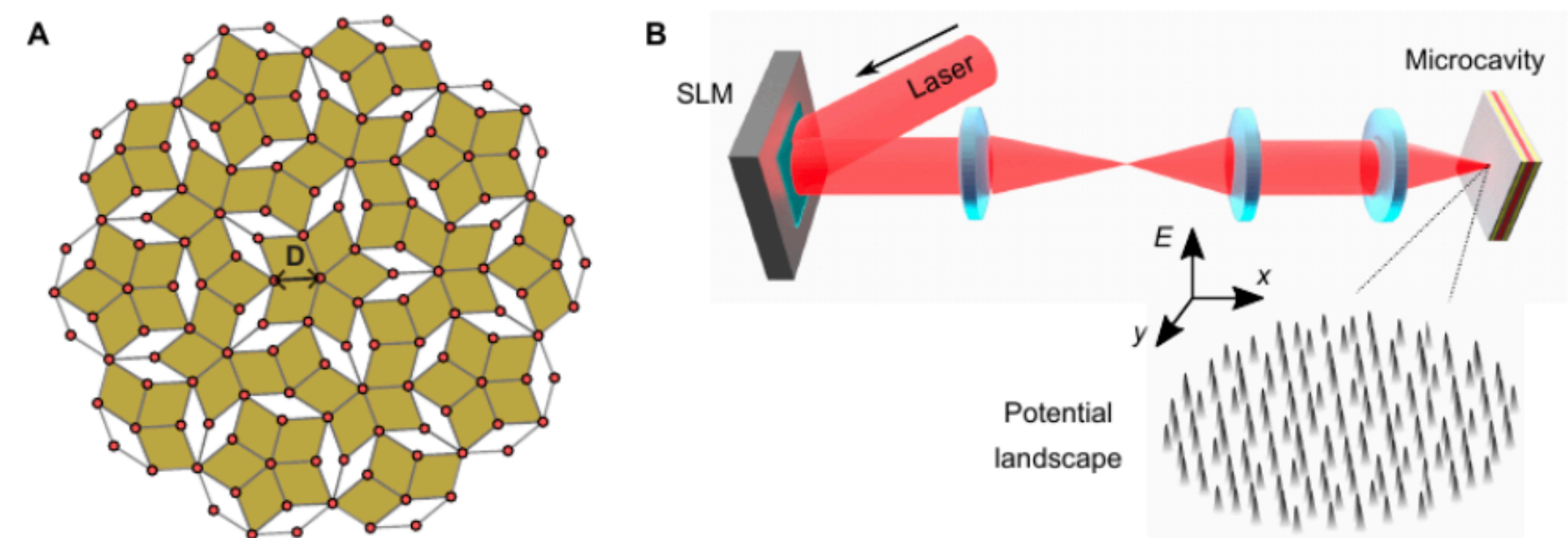
Mukherjee, S. et al. Experimental observation of anomalous topological edge modes in a slowly driven photonic lattice. *Nat. Commun.* 8, 13918 (2017).



Fan-out and interface section



Meany, T. et al. Laser written circuits for quantum photonics. *Laser Photon. Rev.* 9, 363–384 (2015)





**Спасибо за внимание!**